SEARCH RESEARCH:

CURRENT STATE OF KNOWLEDGE ON THE TARGET PROBLEM

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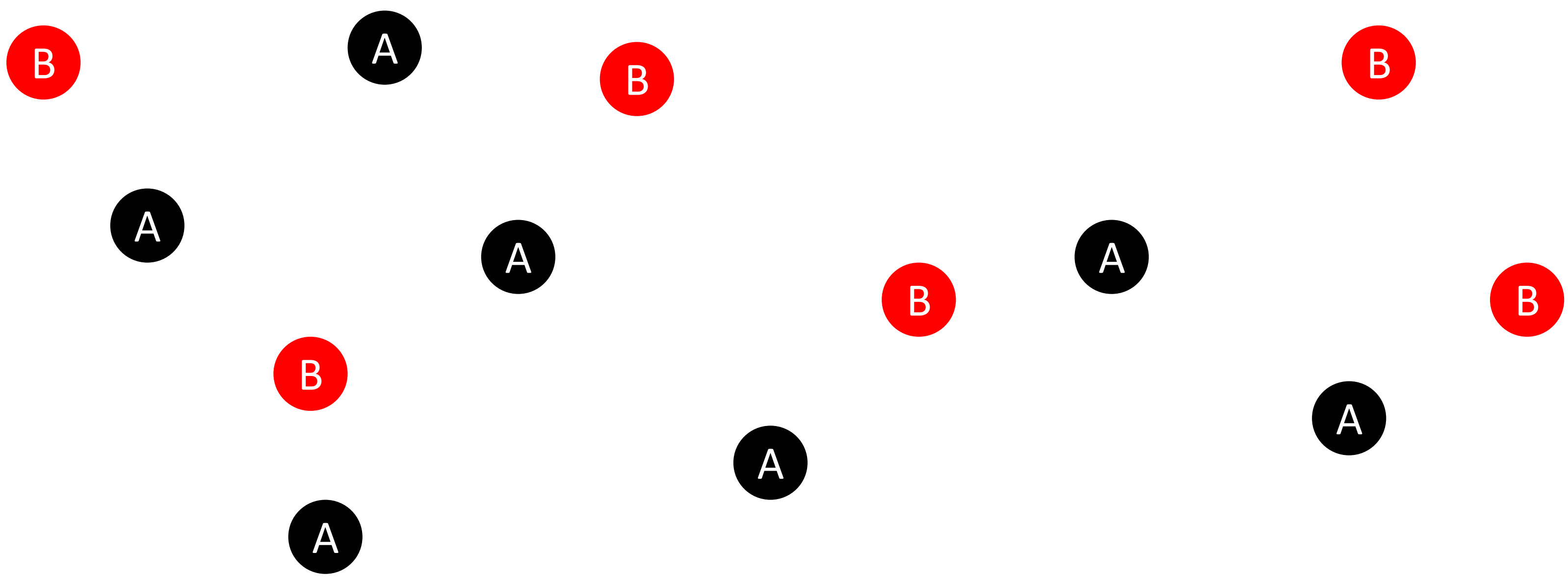
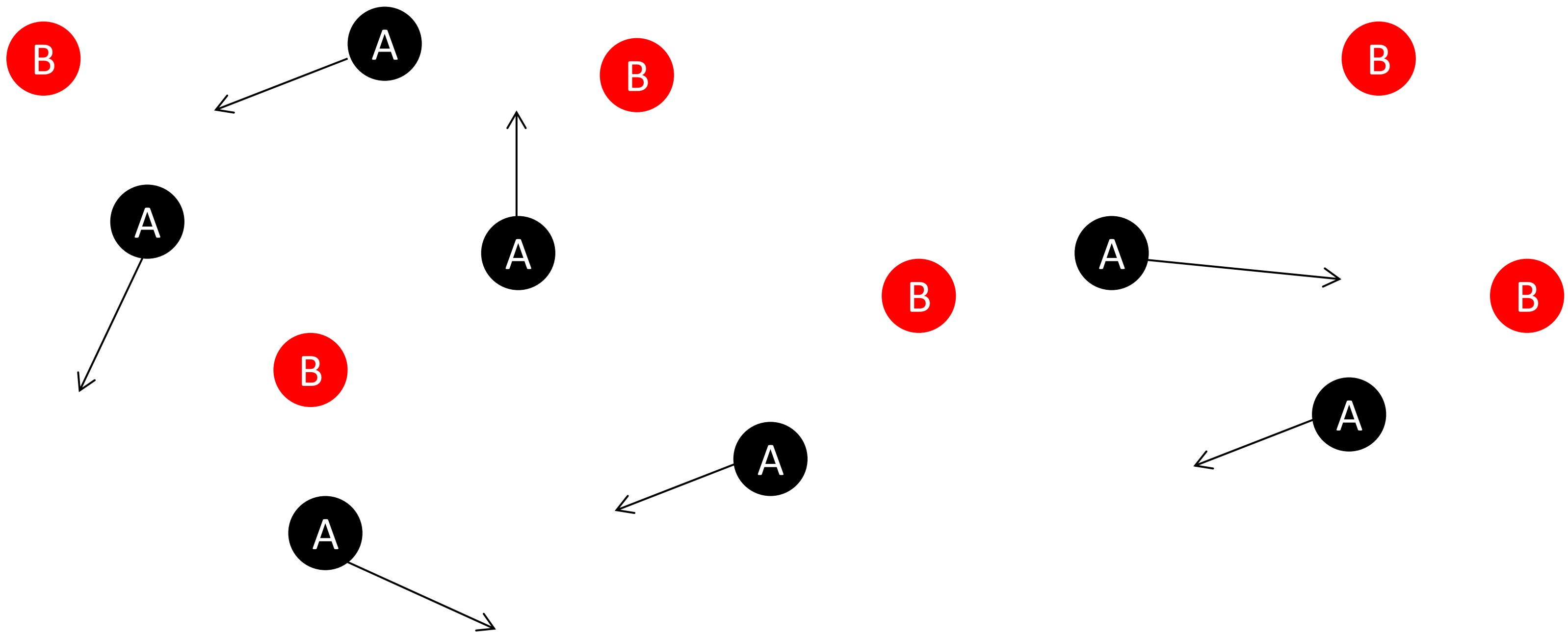
1. Introduction
2. The target problem: the “classics”
3. The “Lévy flight paradigm”
4. Constraints in the target problem
5. Global optima for random search strategies
6. Discussion: the target problem *vs* real searches

1. INTRODUCTION

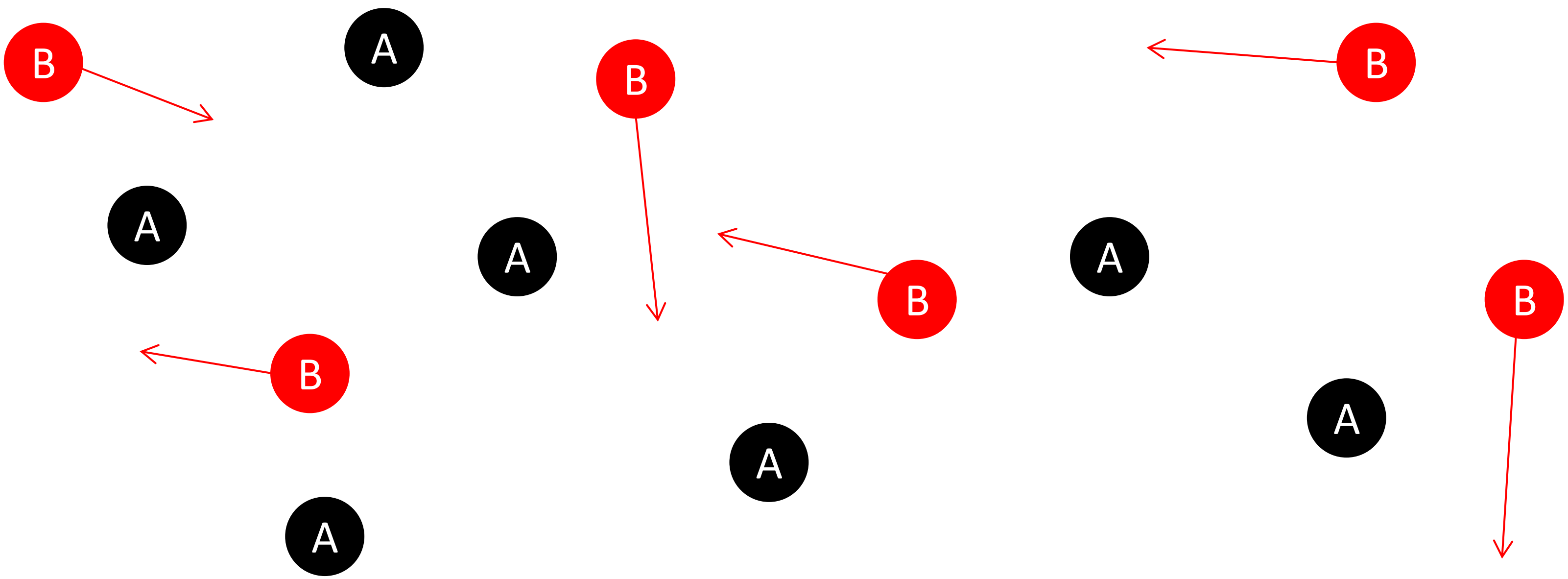


We will describe the position of the i-th particle as a stochastic process $X_i(t)$.

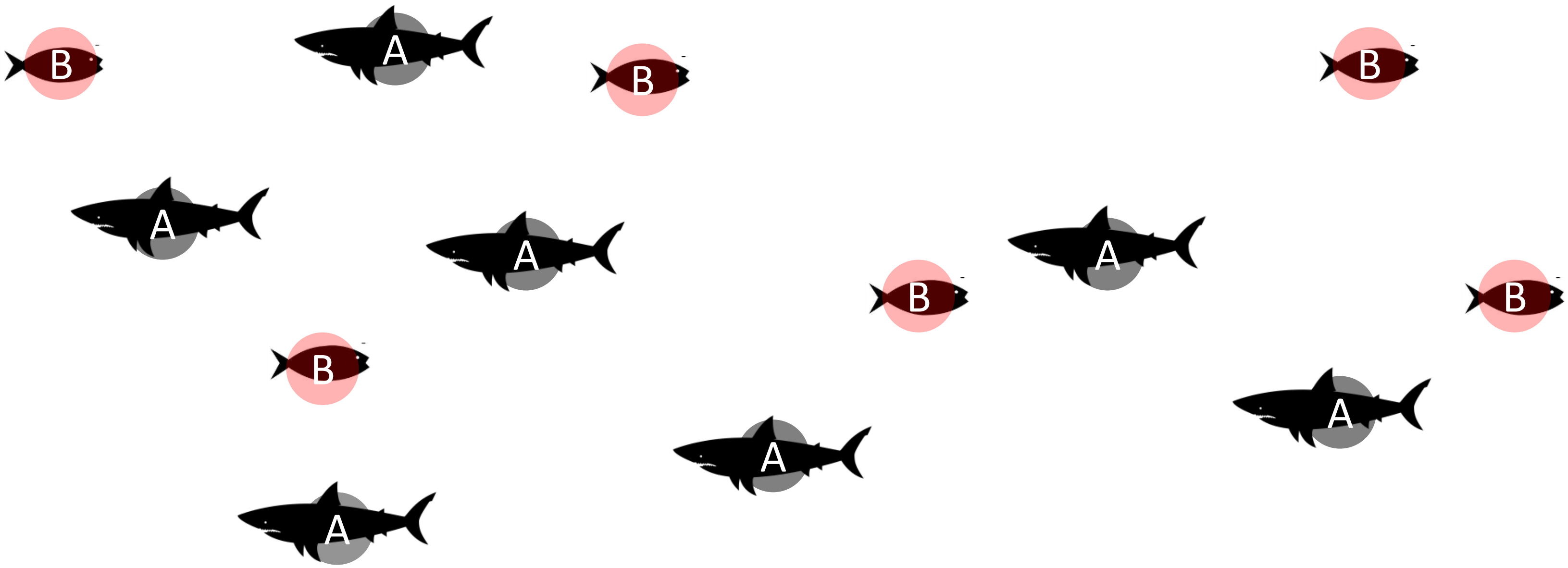
The target problem:



The trapping problem:



Search/foraging analogy (quasi-nule-information context):

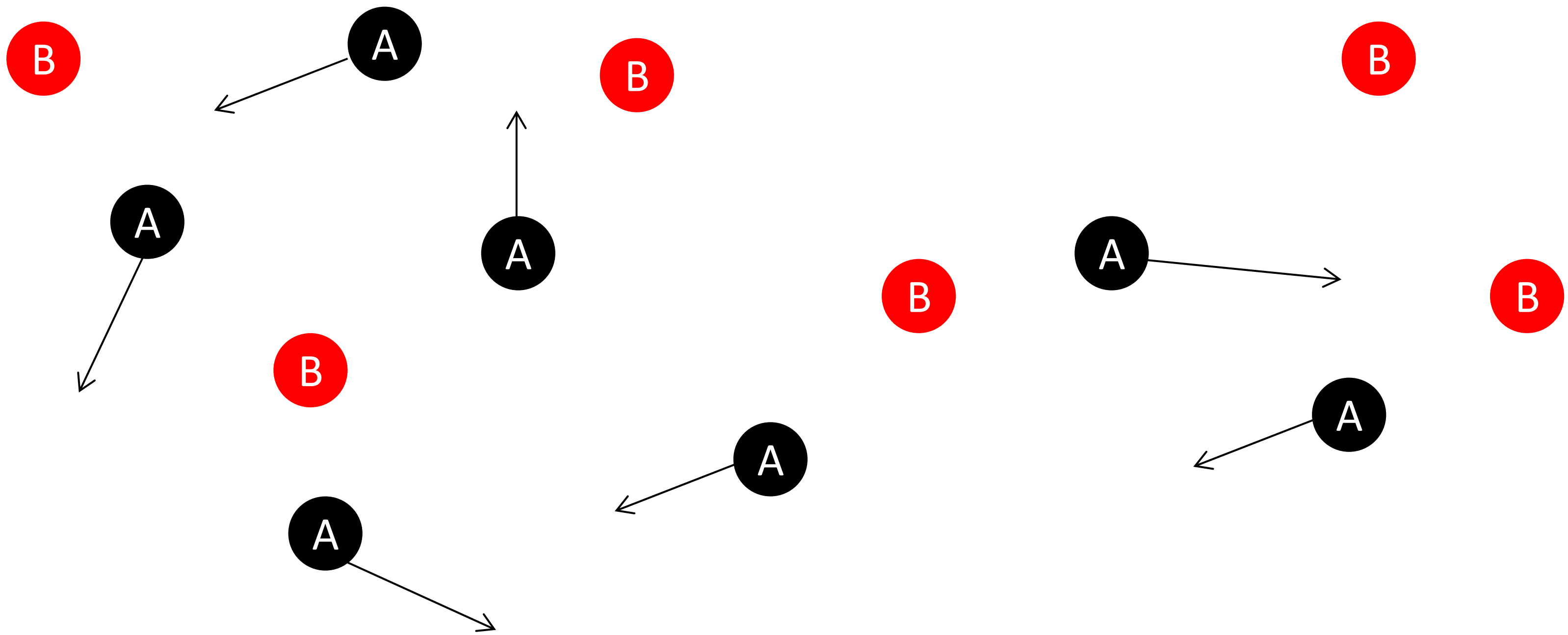


How do we measure search efficiency?

i) Capture rate: $k(t)$
ii) Time distribution to the next capture: $f(t)$
iii) Mean time to the next capture: $\langle T\rangle = \int_0^\infty t f(t) dt$
iv) Survival probability of the prey up to time t_m: $S(t_m)$

Variations of the target problem (I): THE "IDEAL GAS" RANDOM ENCOUNTER HYPOTHESIS

For densities of B high enough:



A **Stationary stochastic process** is defined as that whose joint probability density distribution satisfies $p(x_{t1}, x_{t2}, \dots, x_{tn}) = p(x_{t1+\tau}, x_{t2+\tau}, \dots, x_{tn+\tau})$ for any τ.

Variations of the target problem (II): THE "RANDOM SEARCH" PROBLEM

For densities of A and B low enough:

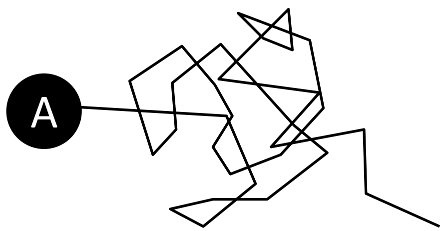
Non-stationary conditions

Variations of the target problem (III): THE “NARROW ESCAPE” PROBLEM

For densities of A and B low enough:

Space-continuous types of motion (I): 'Pure' diffusion model

A **Wiener process** $W(t)$ is defined as a stationary process whose increments $W(t_2) - W(t_1)$ follow a Gaussian distribution with zero mean and variance $|t_2 - t_1|$.



If we assume that $X(t) = x_0 + \sqrt{2D}W(t)$ then:

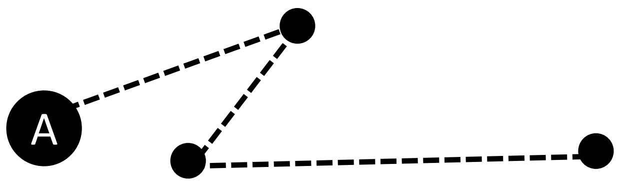
i) The probability density $p(x,t)$ follows a Gaussian distribution with $\langle X \rangle = x_0$ and $\langle X^2 \rangle = 2Dt + {x_0}^2$
ii) It becomes impossible to define a characteristic speed for A (limitations of the *random-walk analogy*)
iii) The problem of infinite propagation signals emerge

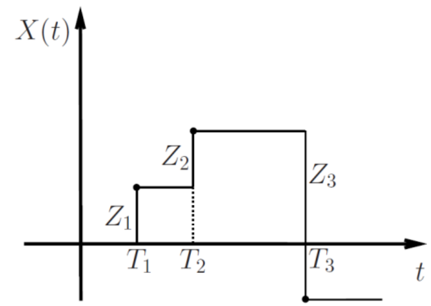
…but the advantage is that we can describe $X(t)$ as a Gaussian (stable) process.

Space-continuous types of motion (II): 'Jump' model

We define the position of the particle after n jumps as: $X_n = \sum_{i=1}^{n} Z_i$

…and the time it takes to perform these n jumps as: $T_n = \sum_{i=1}^{n} \Theta_i$





…where Z_i and Θ_i each are iid random variables distributed, respectively, according to

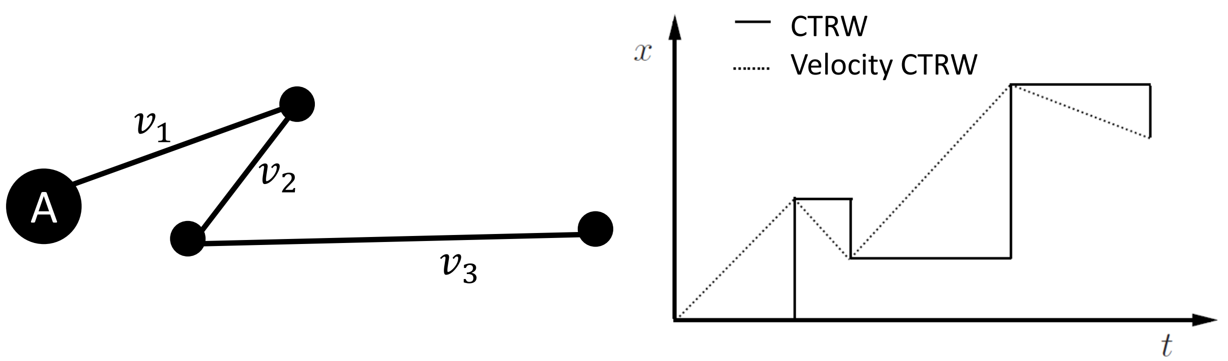
$\phi(x)$: Jump-length probability distribution function (*dispersal kernel*)
$\varphi(t)$: Waiting-time probability distribution function

(This is typically known as a Time-Continuous Random Walk, CTRW)

Diffusive asymptotic limit: $\lim_{t\to\infty} \langle X^2 \rangle = 2\,\frac{\langle Z^2 \rangle}{2d\langle \Theta \rangle}\, t$

Space-continuous types of motion (II): ‘Velocity’ model

We use the same definition as before $X_n = \sum_{i=1}^{n} Z_i$ $T_n = \sum_{i=1}^{n} \Theta_i$



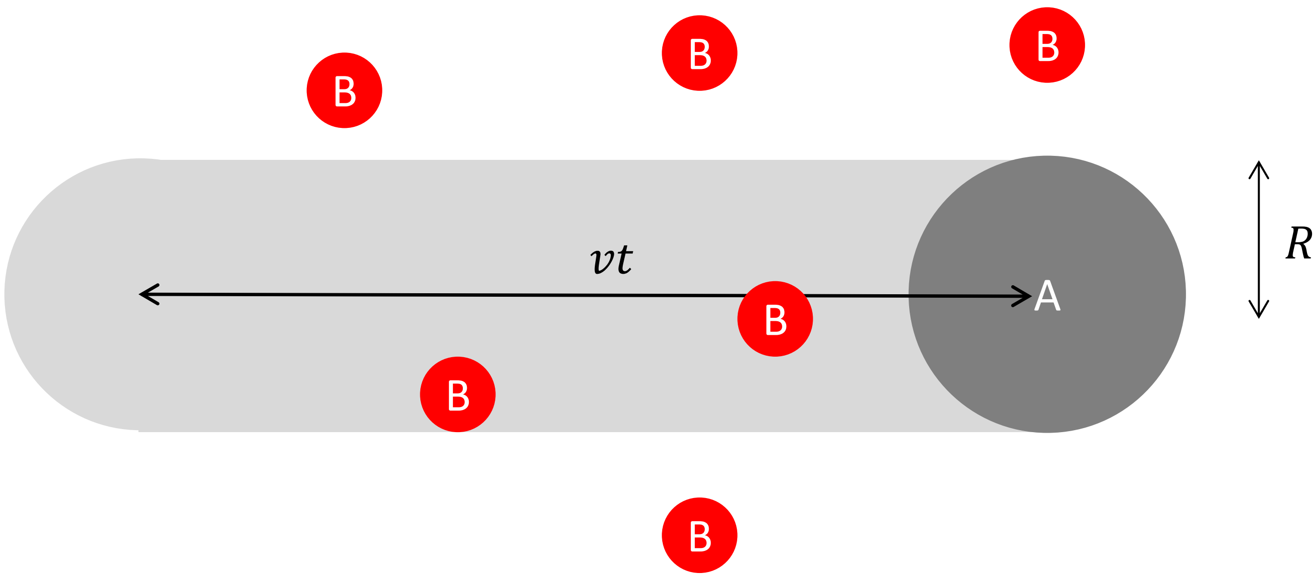
…where now $\varphi(t)$ and $\phi(x)$ are not independent, but coupled through a velocity distribution $h(v)$ in the form

$$\phi(x) = \int_0^{\infty} dt \ \varphi(t) \int_{-\infty}^{\infty} dt \ \delta(x - vt) h(v)$$

(This is typically known as the “velocity version” of the CTRW)

2. THE TARGET PROBLEM: THE “CLASSICS”

THE "IDEAL GAS" RANDOM ENCOUNTER HYPOTHESIS



Due to the stationary assumption, $k(t) = k$

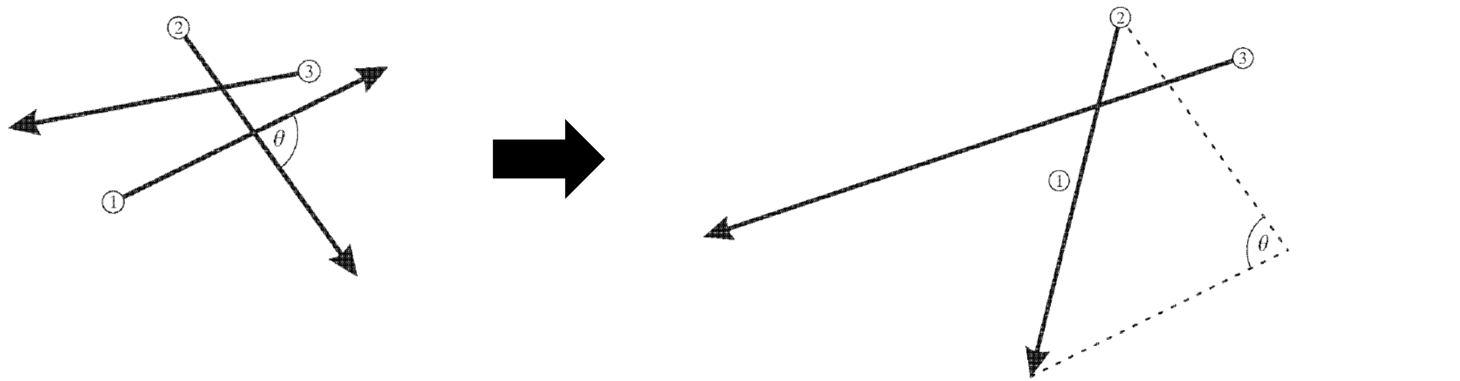
1D: $k = v\rho_B$

2D: $k = 2Rv\rho_B$

3D: $k = \pi R^2 v\rho_B$

THE "IDEAL GAS" RANDOM ENCOUNTER HYPOTHESIS

If we consider only binary-interactions, more general problems can in this case be studied as a target problem:



Relative speed (2D): $\int_0^\infty dv_A \int_0^\infty dv_B \int_0^\pi d\theta p(v_A)p(v_B)\sqrt{{v_A}^2 + {v_B}^2 - 2v_A v_B \cos\theta}$

For fixed speeds v_A and v_B (v_B<v_A):

1D: $k = v_A \rho_B$

3D: $k = \pi R^2 \left(v_A + \frac{{v_B}^2}{3v_A}\right)\rho_B$

For Maxwell-Boltzmann distributions:

2D: $k = 2R\sqrt{\overline{v_A}^2 + \overline{v_B}^2}\rho_B$

3D: $k = \pi R^2\sqrt{\overline{v_A}^2 + \overline{v_B}^2}\rho_B$

THE "IDEAL GAS" RANDOM ENCOUNTER HYPOTHESIS

Connection to other measures of search efficiency:

Mean time between captures:
We use **Kac's recurrence theorem** (which states that, under stationary conditions, the mean time between occurrences of an event equals the inverse of the probability rate to see that event) to show that

$$\langle T \rangle = \frac{1}{k}$$

Time distribution between captures:
The only probability function which is compatible with a constant rate of success (memoryless condition) is an exponential

$$f(t) = ke^{-kt}$$

Survival probability up to t_m:
By definition, $S(t_m) = \int_{t_m}^{\infty} dt f(t)$, so $f(t) = -\left.\frac{dS(t_m)}{dt_m}\right|_t$ and $\langle T \rangle = \int_0^{\infty} dt_m S(t_m)$

This leads to

$$S(t_m) = e^{-kt_m}$$

THE "IDEAL GAS" RANDOM ENCOUNTER HYPOTHESIS

In order to extend the previous results to more realistic (non-ballistic) patterns of motion:

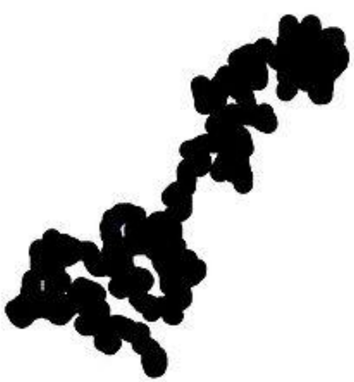
First guess: the diffusive condition $\langle X^2 \rangle = 2Dt$ tells you the average area covered per unit time by the searcher

$$\text{1D:} \quad k(t) \sim \frac{\sqrt{\langle X^2 \rangle}}{t} \rho_B = \frac{2D}{\sqrt{t}} \rho_B$$

$$\text{2D:} \quad k(t) \sim \frac{\langle X^2 \rangle}{t} \rho_B = 2D\rho_B$$

$$\text{3D:} \quad k(t) \sim \frac{\langle X^2 \rangle}{t} 2R\rho_B = 4RD\rho_B$$

Rigurous calculations require computation of the average "Wiener sausage" volume $V(t)$:



$$V(t) = \int d^d\vec{r}\, p^*(\vec{r},t) H(|\vec{r}| - R)$$

For a 'velocity model' in the diffusive regime:

$$\text{1D:} \quad k(t) = \frac{dV(t)}{dt} \rho_B = v \frac{1}{\sqrt{\pi t}} \rho_B$$

$$\text{2D:} \quad \lim_{t \to \infty} k(t) = \frac{4v}{\ln(1.26t)} \rho_B$$

$$\text{3D:} \quad k(t) = V(t)\, \rho_B = v \left(\frac{3}{\sqrt{\pi t}} + 3 \right) \rho_B$$

THE “IDEAL GAS” RANDOM ENCOUNTER HYPOTHESIS

It is also possible to connect the “Wiener sausage” volume to the survival probability:

Non-rigurous proof for a large number N of targets:

For 1 target: $S(t_m) = 1\text{- probability the target is within } V(t)$

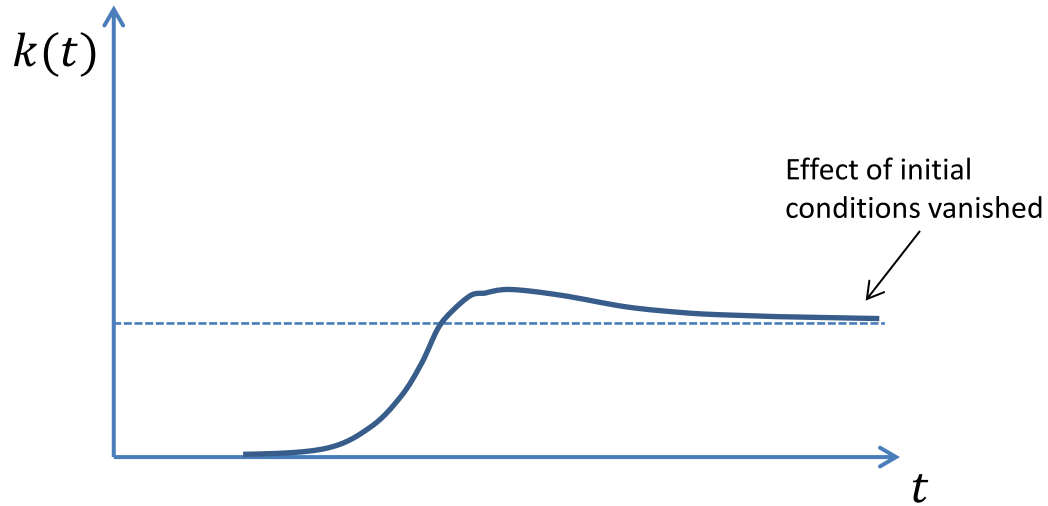
$$S(t_m) = 1 - \frac{\text{number of targets within } V(t)}{\text{total number of targets}}$$

$$S(t_m) = 1 - \frac{V(t)\rho_B}{N}$$

For $N \to \infty$: $S(t_m) = \lim_{N\to\infty}\left(1 - \frac{V(t)\rho_B}{N}\right)^N = exp[-\rho_B V(t)]$

THE “RANDOM SEARCH” PROBLEM

The basic difference lies in the transition to stationary conditions:



In this case the Mean-First Passage Time is usually employed as the most attainable measure of search efficiency. In this situation:

$S(t_m) = \int_{t_m}^{\infty} dt f(t)$ and $\langle T \rangle = \int_0^{\infty} dt_m S(t_m)$ still hold,

but $f(t)$ is not necessarily exponential and $\langle T \rangle \neq \frac{1}{k}$

THE "RANDOM SEARCH" PROBLEM

We formally define the problem as $L_{FP}[p(r,t)] = 0$, with boundary condition $p(\Omega,t) = 0$, being Ω the surface of the target.

$$\boxed{\text{'Pure' diffusion model in 1D:} \quad L_{FP} = \frac{\partial}{\partial t} - D\frac{\partial^2}{\partial x^2}}$$

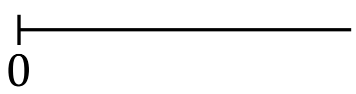
General solution (with $\rho_B = 0$): $p(x,t) = \frac{1}{\sqrt{4\pi D t}} e^{-(x-x_0)^2/4Dt}$

Image method for a target at $x = 0$: $p(x,t) = \frac{1}{\sqrt{4\pi D t}}\left[e^{-(x-x_0)^2/4Dt} - e^{-(x+x_0)^2/4Dt}\right] \approx$

$$\lim_{t\to\infty} p(x,t) \approx \frac{1}{\sqrt{4\pi D t}}\frac{x x_0}{Dt} e^{-(x+x_0)^2/4Dt}$$

$$\rightarrow f(t) = +D\left.\frac{\partial p(x,t)}{\partial x}\right|_0 = \frac{x_0}{\sqrt{4\pi D t^3}} e^{-{x_0}^2/4Dt}$$

$$\rightarrow \int_0^\infty dt f(t) = 1 \quad \text{but} \quad \langle T \rangle \text{ !!!!}$$



$$\rightarrow \quad S(t_m) = \mathrm{erf}\left(\frac{x_0}{\sqrt{4Dt_m}}\right)$$

($f(t) \sim t^{3/2}$: Sparre-Andersen theorem)

THE "RANDOM SEARCH" PROBLEM

'Pure' diffusion model in 1D: $L_{FP} = \frac{\partial}{\partial t} - D\frac{\partial^2}{\partial x^2}$

General solution (with $\rho_B = 0$): $p(x,t) = \sum_{i=1}^{\infty} A_i \sin\left(\frac{i\pi x}{L}\right) e^{-(n\pi/L)^2 Dt}$

Two targets in $x = 0$ and $x = L$:

0 ——— L

$$\lim_{t_m \to \infty} S(t_m) \sim e^{-(\pi/L)^2 D t_m}$$

A closed expression is attainable in the Laplace space $\left(p(x,s) = \int_0^{\infty} dt e^{-st} p(x,t)\right)$

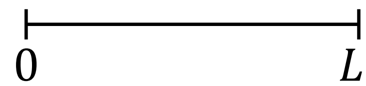
$$p(x,s) = \frac{\sinh\left(\sqrt{\frac{s}{D}}x_0\right)\sinh\left(\sqrt{\frac{s}{D}}(L-x_0)\right)}{\sqrt{sD}\sinh\left(\sqrt{\frac{s}{D}}L\right)}$$

$$\to f(s) = +D\left.\frac{\partial p(x,s)}{\partial x}\right|_0 - D\left.\frac{\partial p(x,s)}{\partial x}\right|_L = \frac{\sinh\left(\sqrt{\frac{s}{D}}x_0\right)\sinh\left(\sqrt{\frac{s}{D}}(L-x_0)\right)}{\sinh\left(\sqrt{\frac{s}{D}}L\right)}$$

$$\to \langle T \rangle = \frac{x_0(L-x_0)}{2D}$$

THE "RANDOM SEARCH" PROBLEM

A method for finding the MFPT $\langle T \rangle$ with targets at $x = 0$ and $x = L$ from L_{FP}:



$$\frac{\partial p(x,t)}{\partial t} = D \frac{\partial^2 p(x,t)}{\partial x^2}$$

$$\frac{\partial p(x_0,t_0)}{\partial t_0} = -D \frac{\partial^2 p(x_0,t_0)}{\partial {x_0}^2}$$ (backward transport equation)

$$\frac{\partial S(t_0)}{\partial t_0} = -D \frac{\partial^2 S(t_0)}{\partial {x_0}^2}$$ (where $S(t_0) = \int_0^L dx p(x,t)$ is the survival probability)

$$-1 = D \frac{\partial^2 \langle T \rangle}{\partial {x_0}^2}$$ (using $\langle T \rangle = \int_0^\infty dt_0 S(t_0)$ as before)

For boundary conditions $\langle T \rangle_{x_0=0} = \langle T \rangle_{x_0=L} = 0$ we get again $\langle T \rangle = \frac{x_0(L-x_0)}{2D}$

THE “RANDOM SEARCH” PROBLEM

‘Velocity’ models in 1D with v fixed and $\varphi(t) = \lambda e^{-\lambda t}$:

Two targets in $x = 0$ and $x = L$:

One needs to separate particles with $+v$ $(p_+(x,t))$ and $-v$ $(p_-(x,t))$

$$L_{FP}[p_+, p_-] = \begin{pmatrix} \frac{\partial}{\partial t} + v\frac{\partial}{\partial x} - \lambda & \lambda \\ \lambda & \frac{\partial}{\partial t} - v\frac{\partial}{\partial x} - \lambda \end{pmatrix}$$

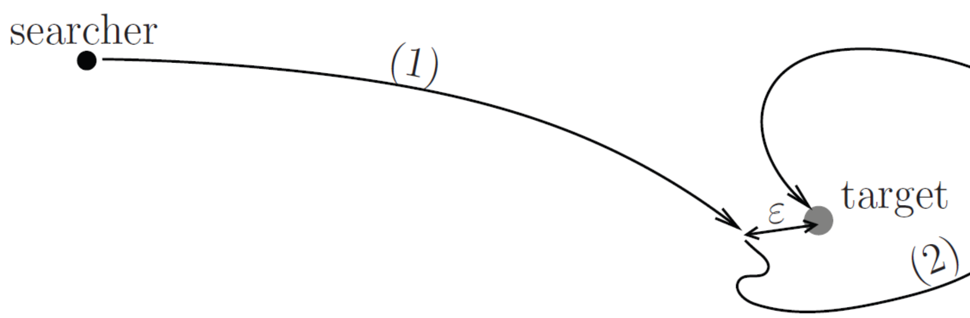
…and introduce boundary conditions $p_-(0,t) = 0$ and $p_+(0,t) = 0$

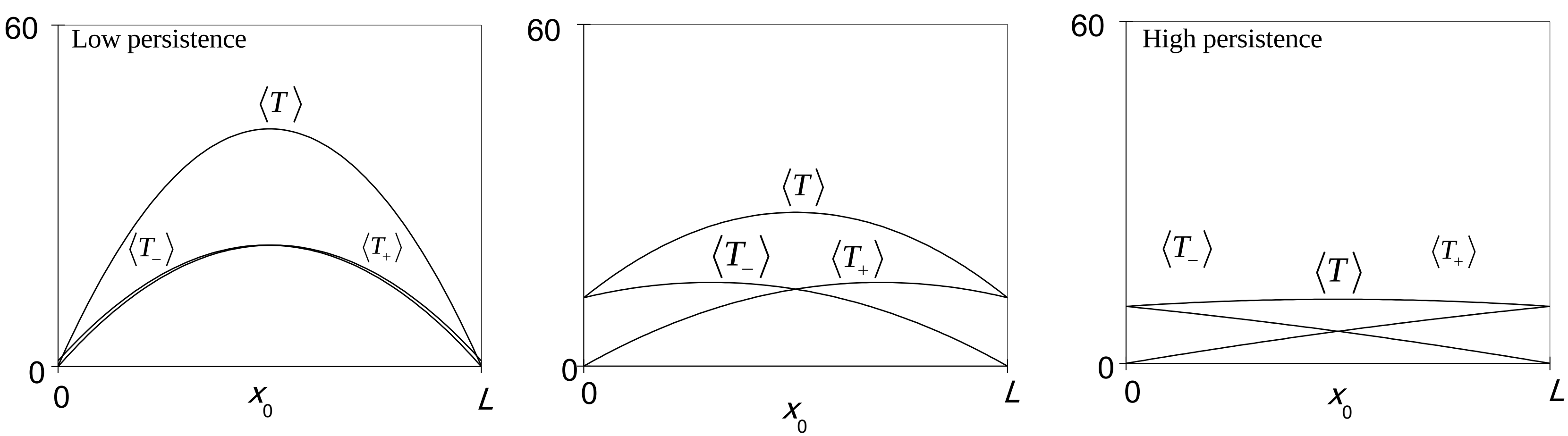
As before, a closed expression is attainable in the Laplace space

$p(x,s) = \frac{1}{2v}\sqrt{\frac{s+\lambda}{s}}\, e^{-\sqrt{s(s+\lambda)}|x-x_0|/v}$ from which $\langle T \rangle = \frac{2x_0(L-x_0)\lambda}{v^2} + \frac{L}{v}$

THE “RANDOM SEARCH” PROBLEM

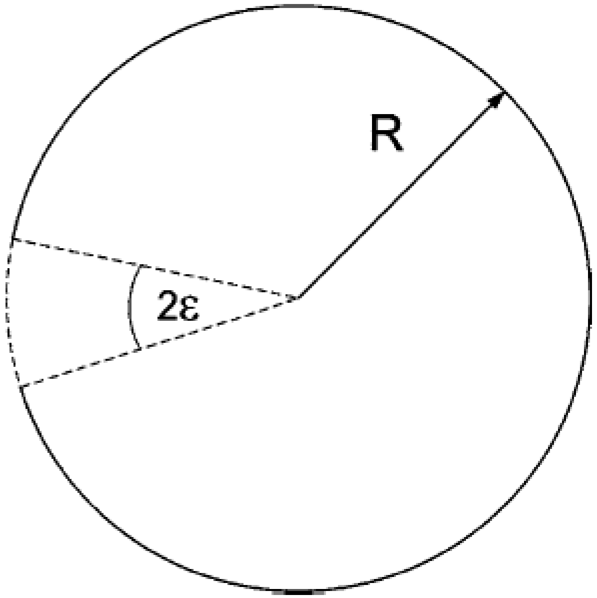
‘Velocity’ models in 1D with v fixed and $\varphi(t) = \lambda e^{-\lambda t}$:





THE “NARROW ESCAPE” PROBLEM

‘Pure’ diffusion model in 2D



Using a method equivalent to that for the MFPT, and working in polar coordinates r, θ we have:

$$D\nabla^2\langle T\rangle = -1 \qquad r < R$$
$$\langle T\rangle = 0 \qquad r = 1, \theta \in \Omega$$
$$\frac{\partial\langle T\rangle}{\partial r} = 0 \qquad r = 1, \theta \in \Omega$$

A solution in the form $\langle T\rangle = \sum_{i=0}^{\infty} A_n r^n \cos i\theta$ is used to obtain:

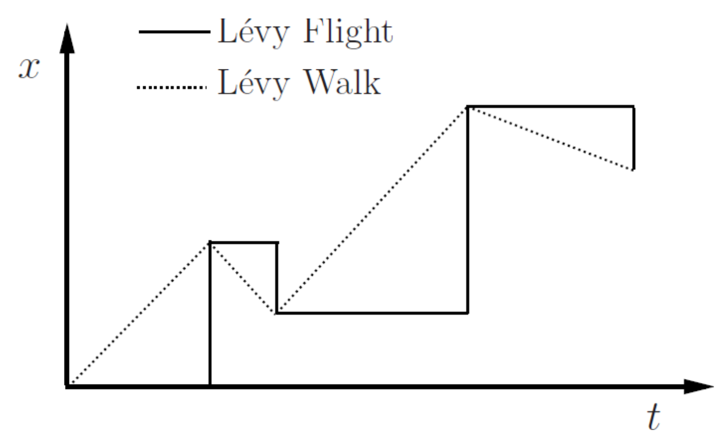
Starting from the center: $\lim_{\varepsilon\to 0}\langle T\rangle = \frac{R^2}{D}\left(\log\frac{1}{\varepsilon} + \frac{1}{4} + \log 2\right)$

In 3D (spherical) domains: $\lim_{\varepsilon\to 0}\langle T\rangle = \frac{\pi R^2}{3D\varepsilon}$

3. THE LEVY FLIGHT PARADIGM

WHAT ARE LEVY FLIGHTS AND LEVY WALKS?

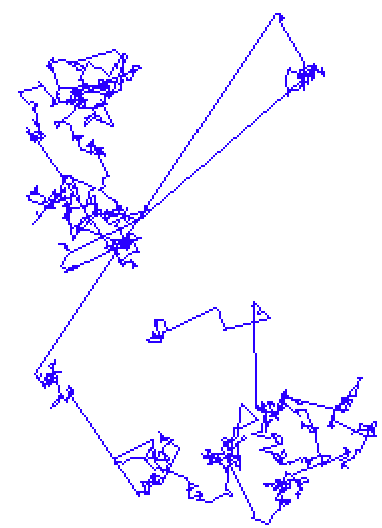
The Lévy Flight fits our ‘jump’ model scheme with $\phi(x)$ a jump length distribution which decays according to $\lim_{t\to\infty} \phi(x) \sim x^{-\mu}$, with $1 < \mu < 3$



The Lévy Walk fits our ‘velocity’ model scheme, with v fixed and $\varphi(t)$ a flight time distribution which decays according to $\lim_{t\to\infty} \varphi(t) \sim t^{-\mu}$, with $1 < \mu < 3$.

Note that this implies that $\langle x^q \rangle \equiv \int_{-\infty}^{\infty} dx \phi(x) x^q$ and $\langle t^q \rangle \equiv \int_0^{\infty} dt \varphi(t) t^q$, respectively, diverge for $q - \mu \geq -1$

In the Lévy Flight case these divergences extend to the overall behavior of the particle, so $\langle X^2 \rangle$ also diverges. In contrast, for the Lévy Walk case, thanks to the coupling between flight durations and lengths through v:



$$\langle X^2 \rangle \sim \begin{cases} t^2 & , 1 < \mu < 2 \\ t^{4-\mu} & , 2 < \mu < 3 \end{cases}$$

(Viswanathan et. al. Nature 401, 911 (1999))

THE LÉVY FLIGHT OPTIMAL SOLUTION

Define the search efficiency $\frac{1}{\langle l \rangle N}$, where $\langle l \rangle$ is the mean flight distance between targets and N the mean number of flights to cover the distance between targets

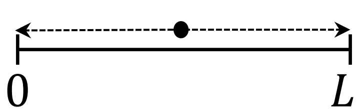
Given $\phi(x) \sim x^{-\mu}$ and a mean path between targets of β,

$$\langle l \rangle \approx \frac{\int_0^\beta dx\, x^{1-\mu} + \beta \int_\beta^\infty dx\, x^{-\mu}}{\int_0^\infty dx\, x^{-\mu}}$$

and the mean number of flights satisfies $N \sim \beta^{(\mu-1)/2}$ if the target is close enough.
All this leads to a search efficiency optimization for $\mu = 2$.

INTUITIVE MEANING

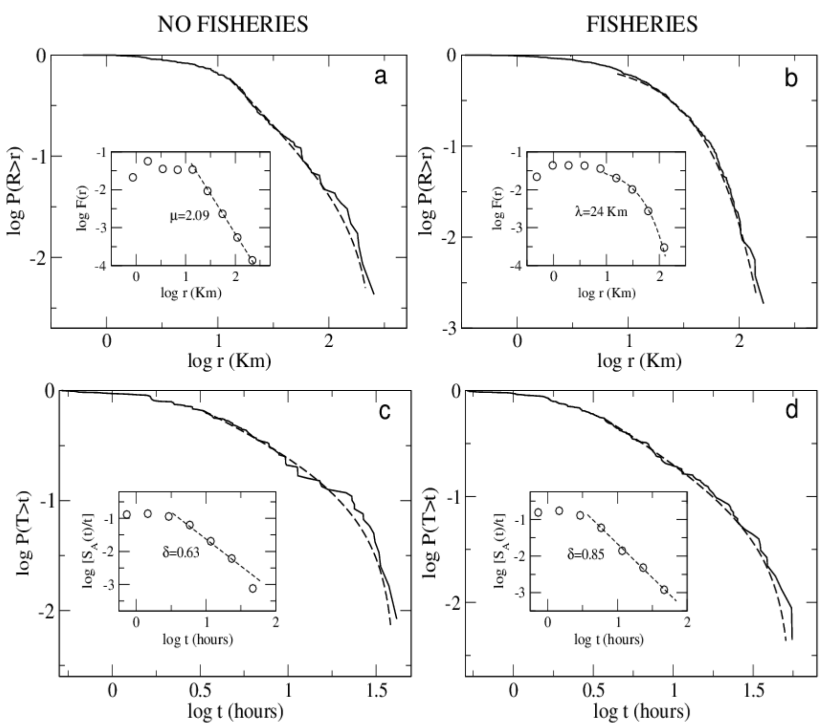
Optimal ballistic approach

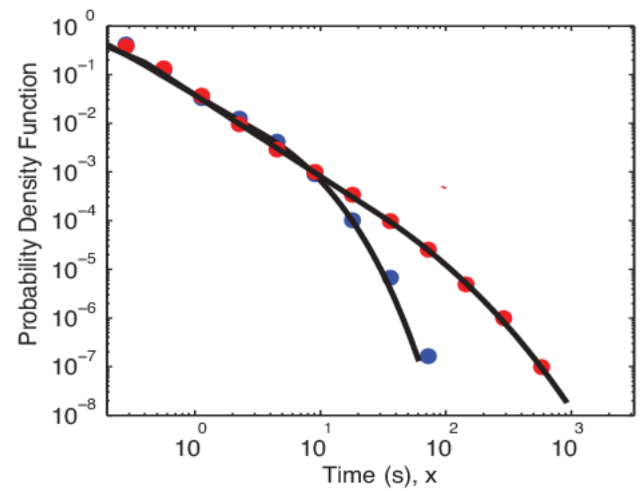


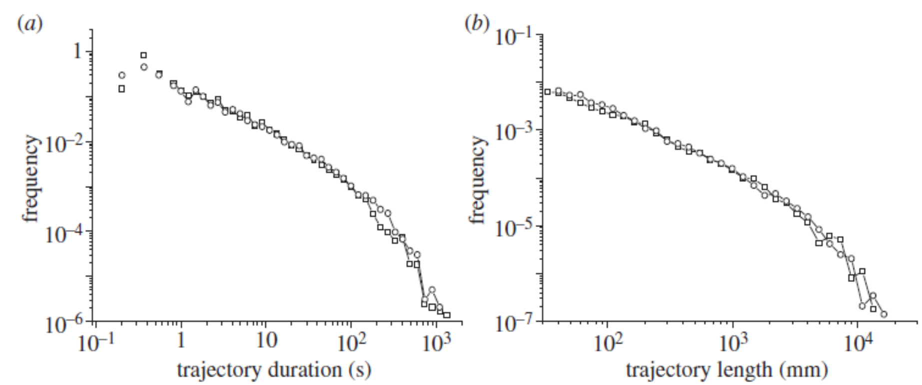
Optimal reorientation ('correction')

THE LEVY FLIGHT FORAGING HYPOTHESIS

"Given Lévy Flight optimality, evolution should have favoured sensorymotor mechanisms that facilitate the emergence of motion patterns similar to the Lévy case in search situations of poor information."







PROBLEMS AND DEBATES AROUND THE LEVY FLIGHT HYPOTHESIS

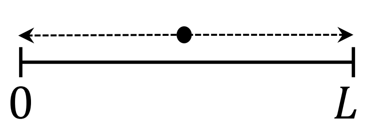
<u>Rigurous experimental verification of Lévy patterns</u>

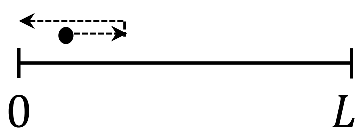
Experimental trajectories, if not analyzed with appropriate statistical methods, can exhibit Lévy patterns; this can happen for: (i) Lévy-Brownian transitions, (ii) inappropriate path sampling, (iii) inappropriate experimental windows (iv) inappropriate fitting techniques, etc. (METHODOLOGICAL QUESTIONS)

<u>Destructive *vs* non-destructive foraging</u>

The result for the mean number of flights change from $N \sim \beta^{(\mu-1)/2}$ to $N \sim \beta^{(\mu-1)}$. Note the analogy with the result for the MFPT with ‘pure’ diffusion

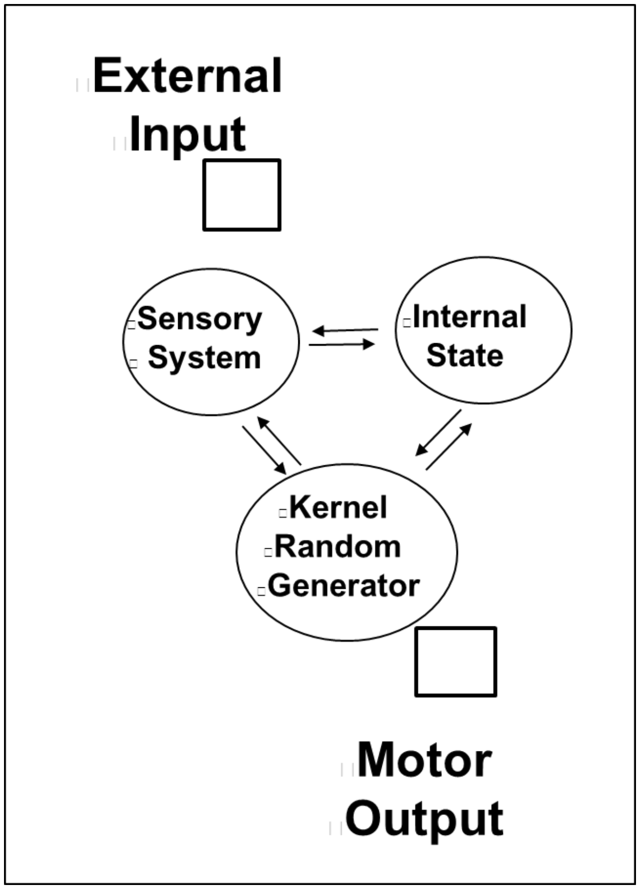
$$\langle T \rangle = \frac{x_0(L - x_0)}{2D}$$

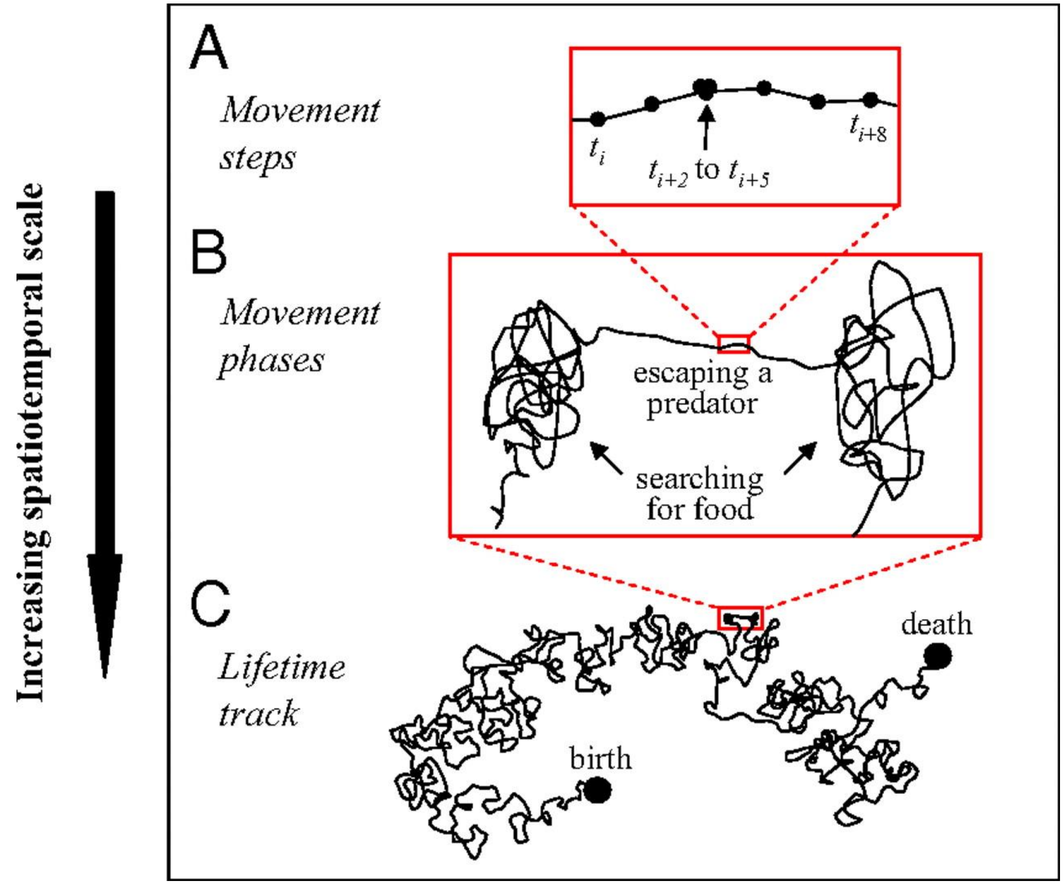




PROBLEMS AND DEBATES AROUND THE LEVY FLIGHT HYPOTHESIS

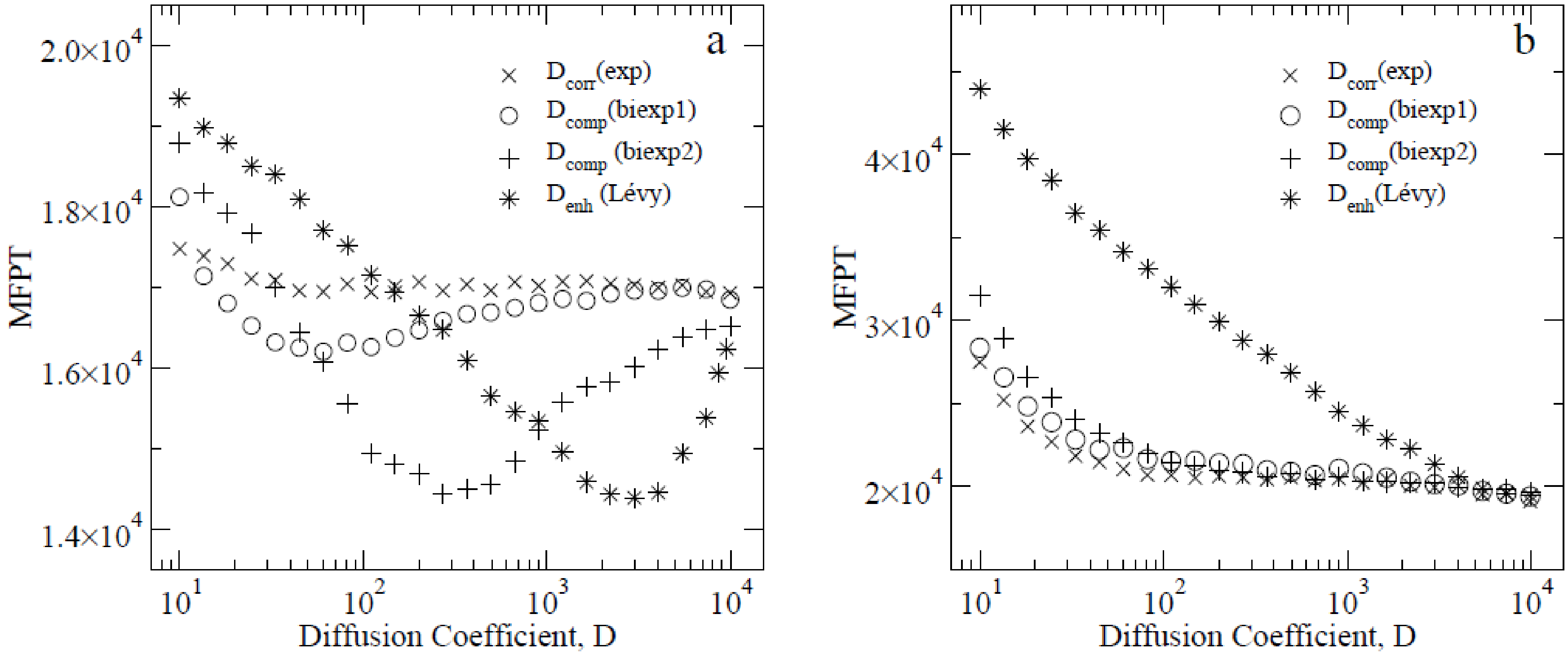
<u>How to design appropriate experiments?</u>





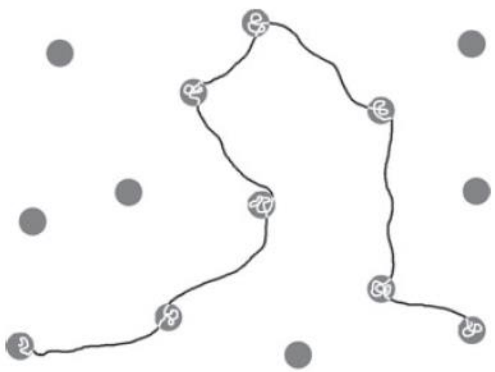
Nathan R et al. PNAS 2008;105:19052-19059

ARE LÉVY FLIGHTS SO SPECIAL?

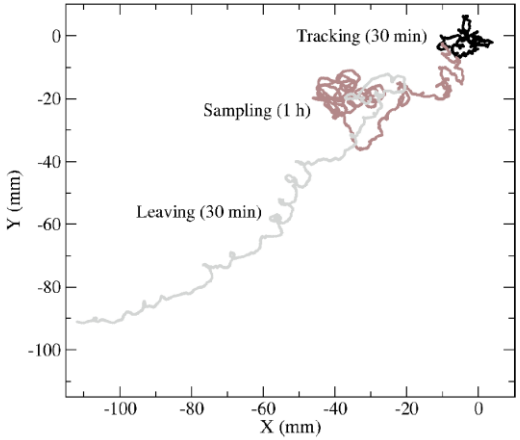


<u>WHAT ARE **REALLY** THE RELEVANT QUESTIONS ABOUT THE LEVY FLIGHT HYPOTHESIS?</u>

a) Can the observed motion patterns be explained through external or internal factors?

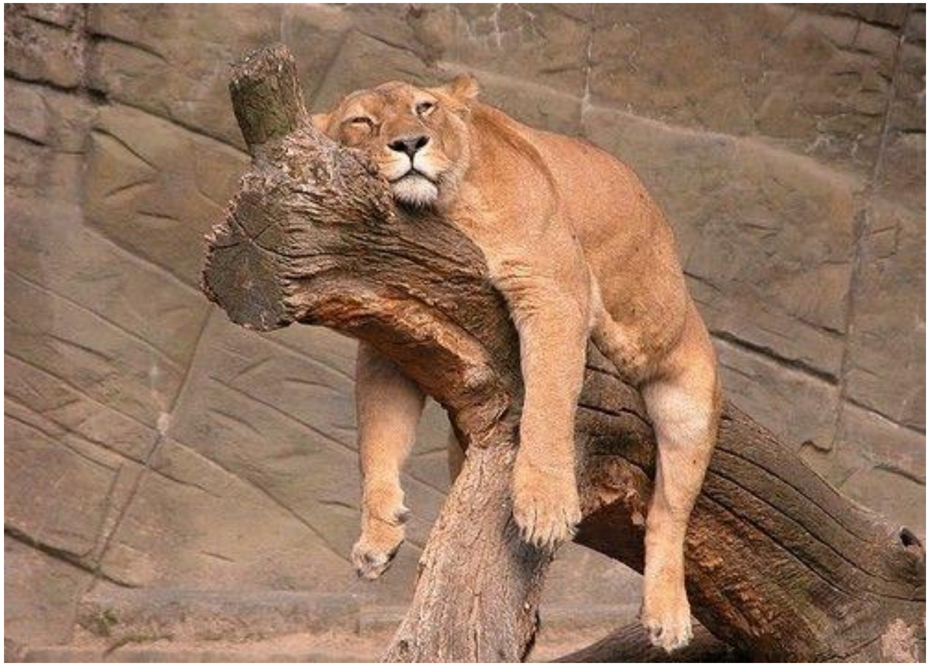


vs



b) Does the emergence of multiple internal scales arise from internal clocks, mental states, multitasks, inter-individuals differences,…?

4. CONSTRAINTS IN THE TARGET PROBLEM



THE GENERALIZED "RANDOM SEARCH" PROBLEM

'Velocity' models

We need a new (more versatile) way to quantify things:

$$k(t)dt = \int_0^{\infty} dv \int_{0-vdt}^{0} dx\, p(x,v,t) + \int_{-\infty}^{0} dv \int_0^{vdt} dx\, p(x,v,t) \approx$$
$$\approx \int_0^{\infty} dv\, vp(0,v,t)dt + \int_{-\infty}^{0} dv\, vp(0,v,t)dt$$

We assume one target located at $x = 0$ and introduce $k_n(t)$ as the rate at which the n-th detection occurs:

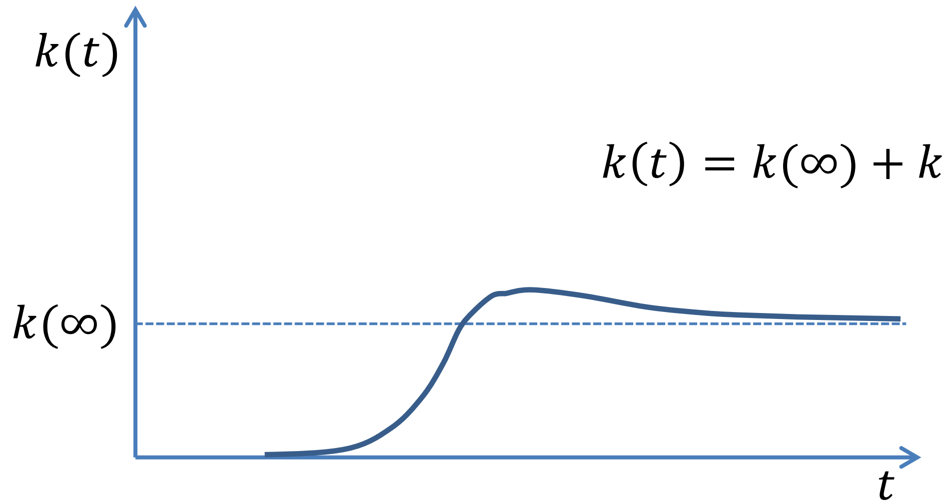
$$k(t) = k_1(t) + k_2(t) + k_3(t) + \cdots = f(t) + f(t) * f_0(t) + f(t) * f_0(t) * f_0(t) + \cdots =$$

$$k(s) = f(s) \sum_{i=0}^{\infty} f_0(s) = \frac{f(s)}{1 - f_0(s)} \quad \Rightarrow \quad \boxed{f(s) = \frac{k(s)}{1 + k_0(s)}}$$

THE GENERALIZED "RANDOM SEARCH" PROBLEM

'Velocity' models

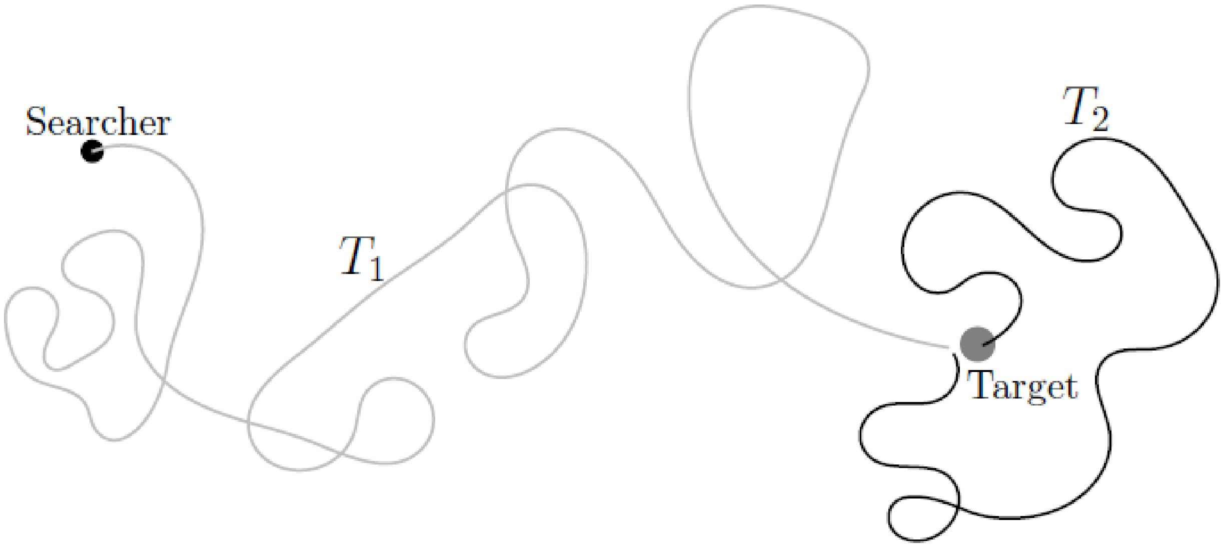
An essential advantage of this framework is that it allows a very general and intuitive understanding of the Mean First Detection Time (MFDT):



$$\Rightarrow \langle T\rangle = \int_0^\infty dt\, t f(t) = \lim_{s\to 0} \frac{df(s)}{ds} = \lim_{s\to 0}\left(\underbrace{\frac{k_0{}^*(s)}{k(\infty)}}_{T_{1a}} - \underbrace{\frac{k^*(s)}{k(\infty)}}_{T_{1b}}\right) + \underbrace{\frac{1}{k(\infty)}}_{T_2}$$

THE GENERALIZED "RANDOM SEARCH" PROBLEM

'Velocity' models



$T_1 = T_{1a} + T_{1b}$	$\rightarrow$	Approach time
T_2	$\rightarrow$	Return time
$T_{1a} + T_2$	$\rightarrow$	Homogenous initial conditions

RANDOM SEARCHES WITH “MORTAL” PARTICLES

For a constant mortality rate ω:

$$k(t) \to e^{-\omega t} k(t) \qquad \to \qquad k(s) \to k(s+\omega) \qquad \to \quad f(s) = \frac{k(s+\omega)}{1+k_0(s+\omega)}$$

Here $S(\infty)$ is the most relevant parameter:

$$S(\infty) = \int_0^\infty dt\, f(t) = \lim_{s\to 0} \int_0^\infty dt\, e^{-st} f(t) = \lim_{s\to 0} f(s)$$

For the ‘pure’ diffusion model:

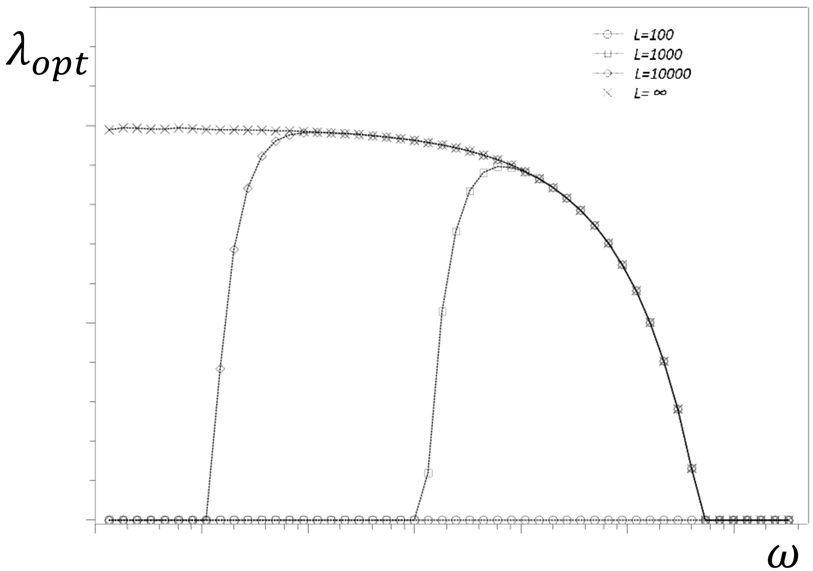
$$S(\infty) = 1 - \frac{\sqrt{\omega\lambda}\left(e^{-\sqrt{\omega\lambda}\, x_0/v} + e^{-\sqrt{\omega\lambda}\,(L-x_0)/v}\right)}{1 + e^{-\sqrt{\omega\lambda}\, L/v}}$$

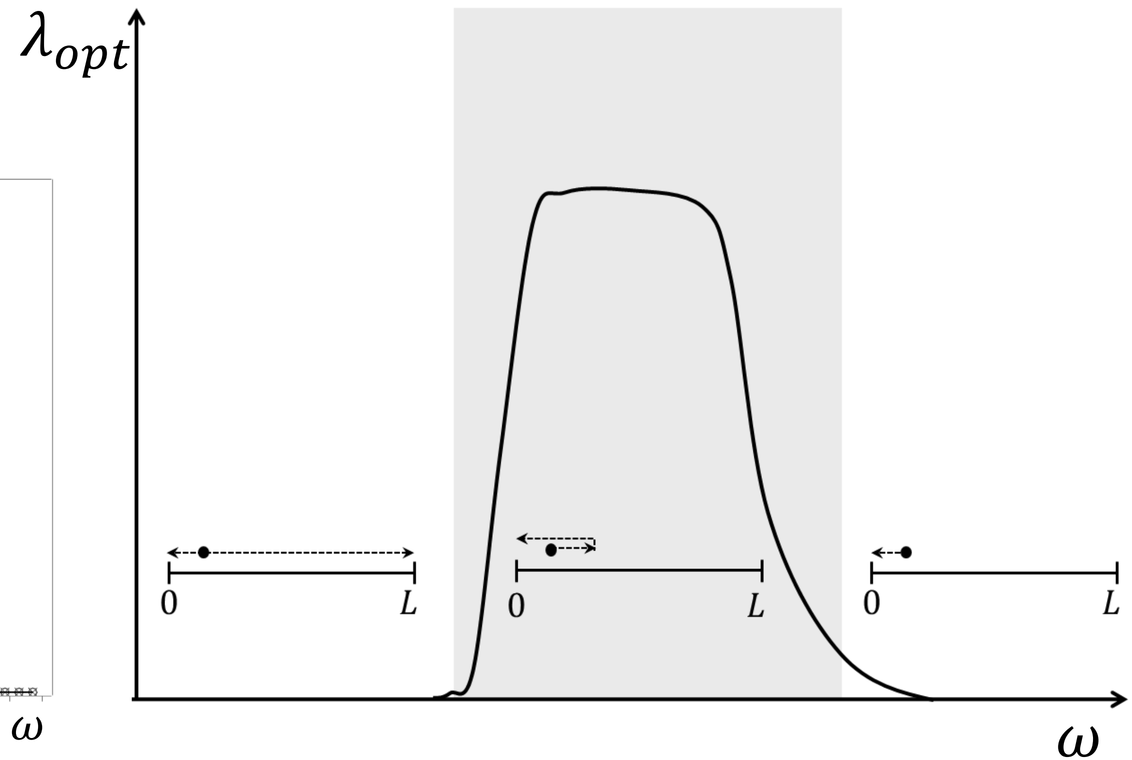
For the ‘velocity’ model with v fixed and $\varphi(t) = \lambda e^{-\lambda t}$:

$$S(\infty) = 1 - \frac{\sqrt{\omega(\omega+\lambda)}\left(e^{-\sqrt{\omega(\omega+\lambda)}x_0/v} + e^{-\sqrt{\omega(\omega+\lambda)}(L-x_0)/v}\right)}{\omega\left(1 - e^{-\sqrt{\omega(\omega+\lambda)}L/v}\right) + \sqrt{\omega(\omega+\lambda)}\left(1 + e^{-\sqrt{\omega(\omega+\lambda)}L/v}\right)}$$

RANDOM SEARCHES WITH “MORTAL” PARTICLES

Implications on the Lévy flight paradigm:





RANDOM SEARCHES WITH NON-PERFECT DETECTION

For a constant probability α of detection, $k(t) \to \alpha k(t)$

For a speed-dependent probability $\alpha = \alpha(v)$:

$$k(t)dt = \int_0^\infty dv \int_{0-vdt}^0 dx\, \alpha(v)p(x,v,t) + \int_{-\infty}^0 dv \int_0^{vdt} dx\, \alpha(v)p(x,v,t) \approx$$
$$\approx \int_0^\infty dv\, v\alpha(v)p(0,v,t)dt + \int_{-\infty}^0 dv\, v\alpha(v)p(0,v,t)dt$$

In general, $\alpha = \alpha(x,v,t)$:

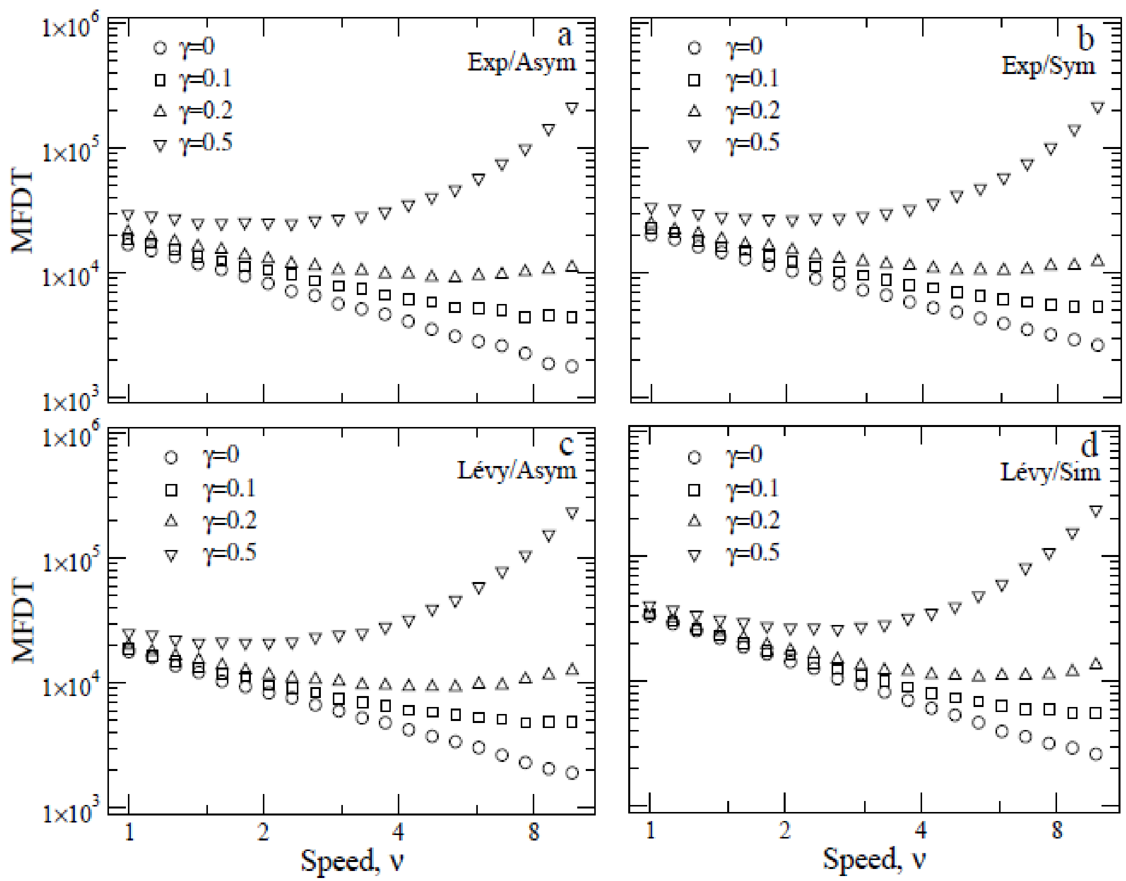
$$k(t)dt \approx \int_0^\infty dv\, v\alpha(x,v,t)p(0,v,t)dt + \int_{-\infty}^0 dv\, v\alpha(x,v,t)p(0,v,t)dt$$

RANDOM SEARCHES WITH NON-PERFECT DETECTION

Case 1D 'velocity' model with v fixed and $\varphi(t) = \lambda e^{-\lambda t}$, $\alpha = e^{-\gamma v}$

$$\langle T \rangle = \underbrace{\frac{2x_0(L-x_0)\lambda}{v^2}}_{\boldsymbol{T_1}} + \underbrace{\frac{L}{v\alpha(v)}}_{\boldsymbol{T_2}}$$

T_1 independent of α

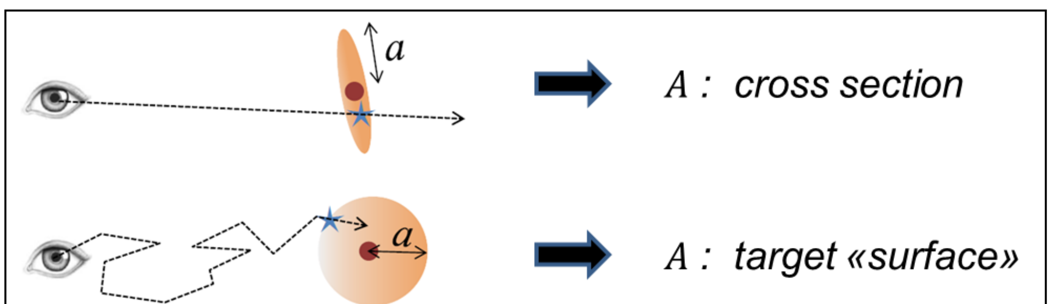


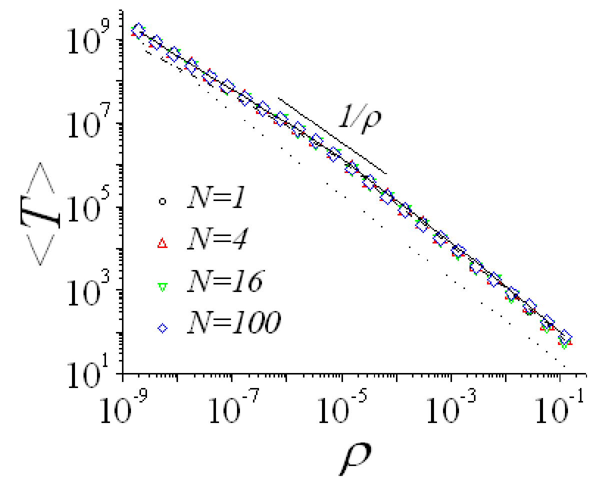
RANDOM SEARCHES WITH NON-PERFECT DETECTION

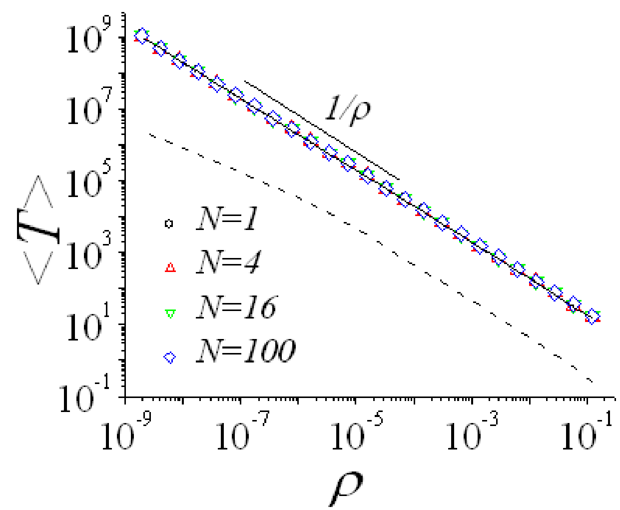
Case 2D or higher 'velocity' model with v fixed and $\varphi(t) = \lambda e^{-\lambda t}$, $\alpha = \alpha(v)$

$$\langle T \rangle = \underbrace{\frac{L^2}{v^2} g_d(x_0)}_{\boldsymbol{T_1}} + \underbrace{\frac{L^d}{Av\alpha(v)}}_{\boldsymbol{T_2}}$$

$$\langle T \rangle = \frac{L^2}{v^2} g_d(x_0) + \frac{1}{\rho}\frac{1}{Av\alpha(v)}$$

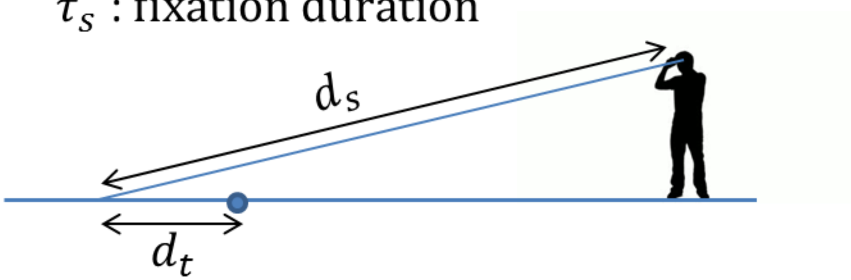


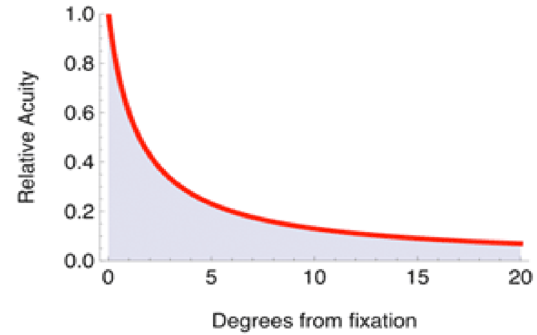




NON-PERFECT DETETION: SACCADE-FIXATION MECHANISM

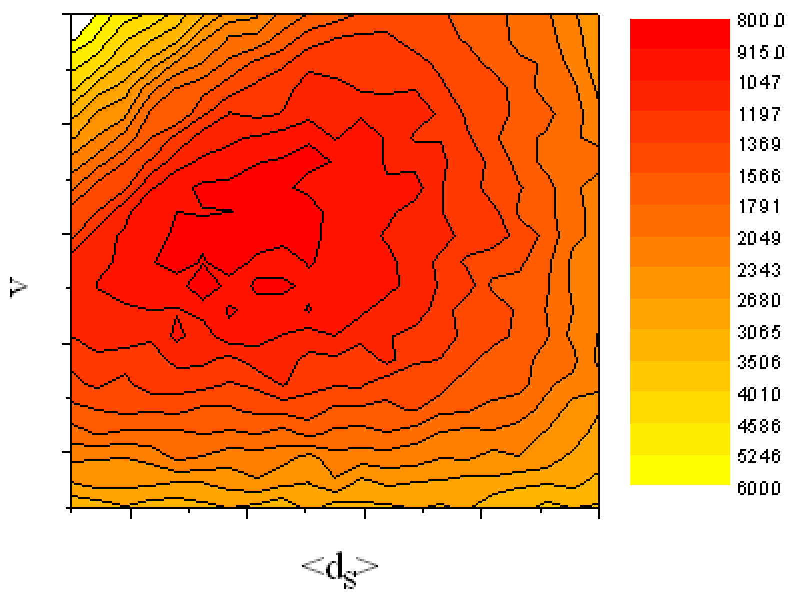
τ_s : fixation duration



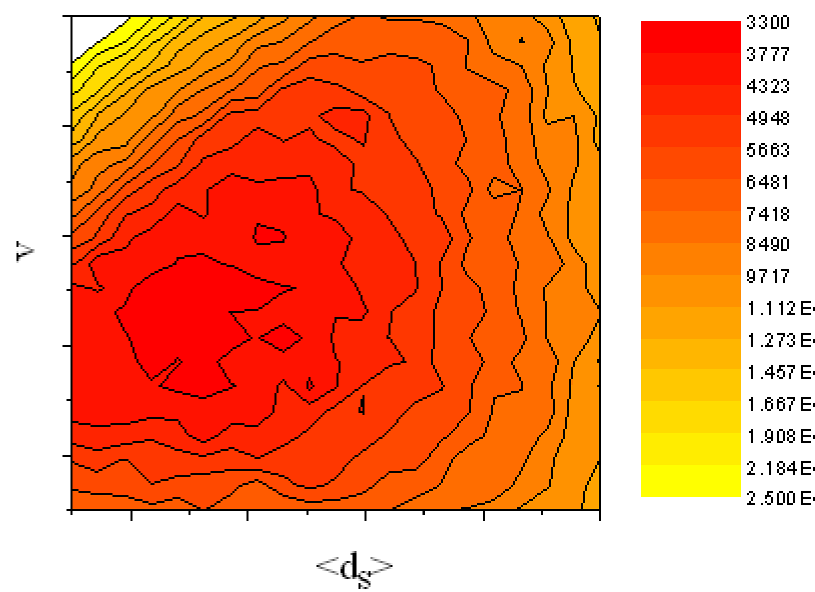


$$\boxed{\alpha = c\tau_s {d_s}^{-2} {d_t}^{-1}}$$

$c = 1$

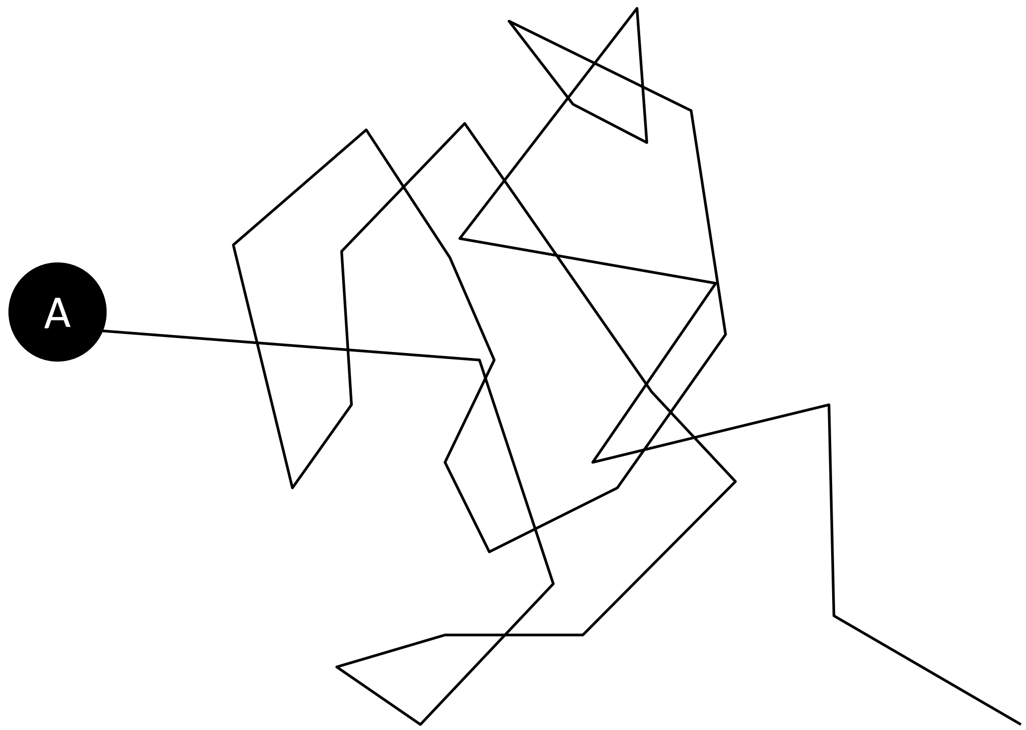


$c = 0.2$



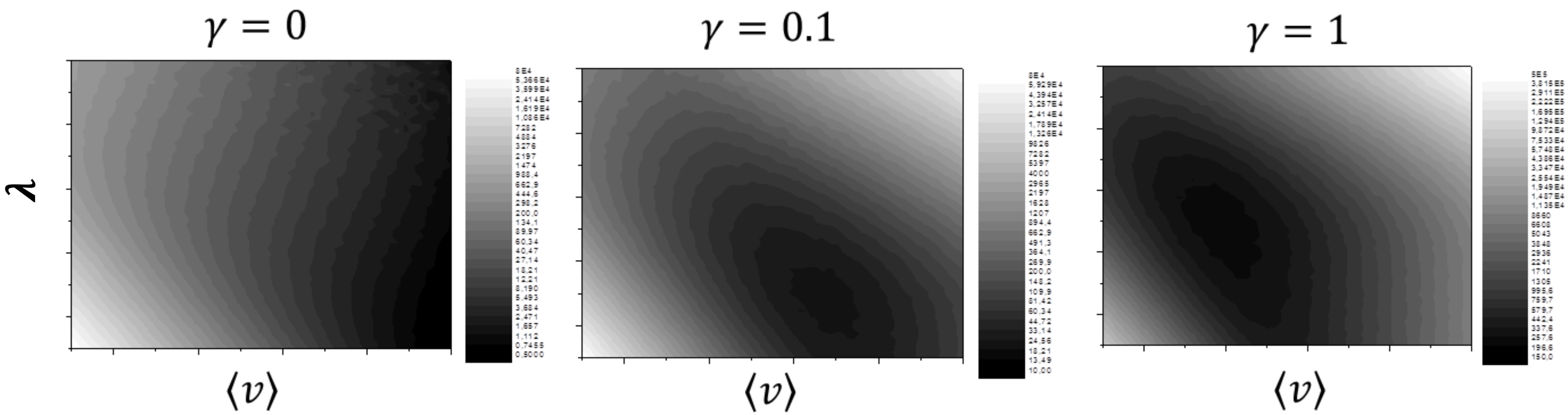
5. GLOBAL OPTIMA FOR RANDOM SEARCH STRATEGIES

WHAT ARE THE DEGREES OF FREEDOM OF A RANDOM SEARCHER?



The optimization parameters/strategies are $v, \varphi(t)$ and, partially, $\alpha(v)$.

Case 2D or higher 'velocity' model with v fixed and $\varphi(t) = \lambda e^{-\lambda t},\ \alpha = e^{-\gamma v}$



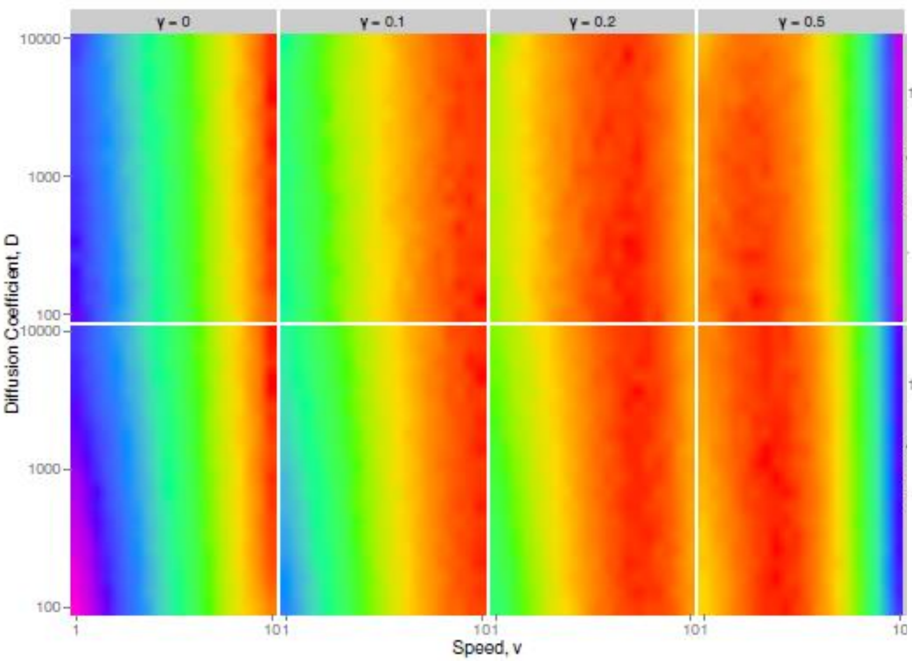
For energetic constraints, we will find a similar effect. Defining a metabolic consumption rate $\alpha_e(v,t)$, minimizing consumption means

$$\langle E \rangle = \left\langle \int_0^T dt \alpha_e(v,t) \right\rangle$$

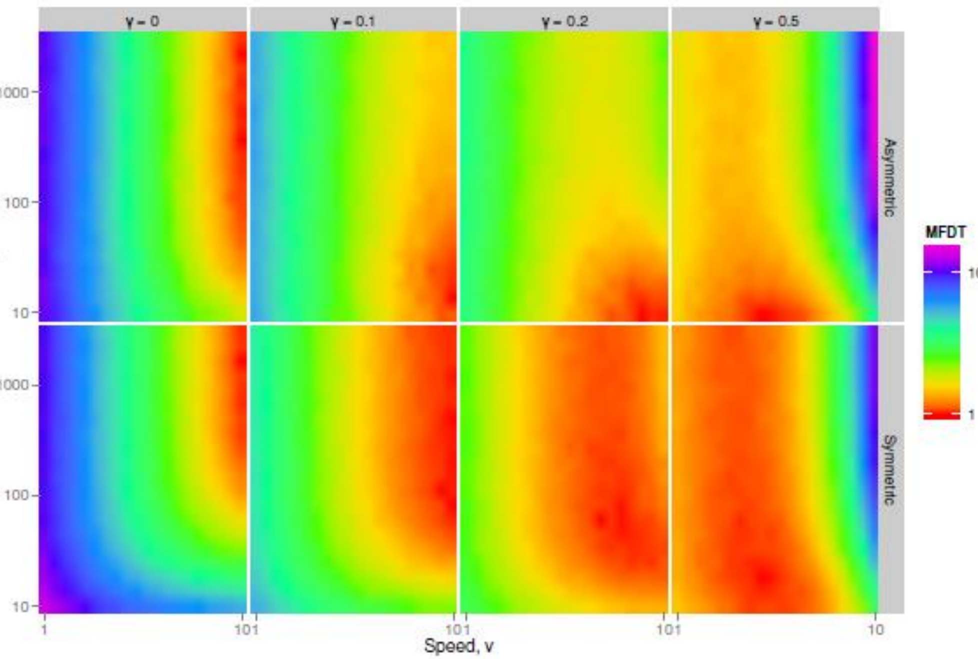
so minimizing the MFDT and minimizing energy are equivalent in the time-independent case.

EXPONENTIAL vs LÉVY GLOBAL OPTIMA

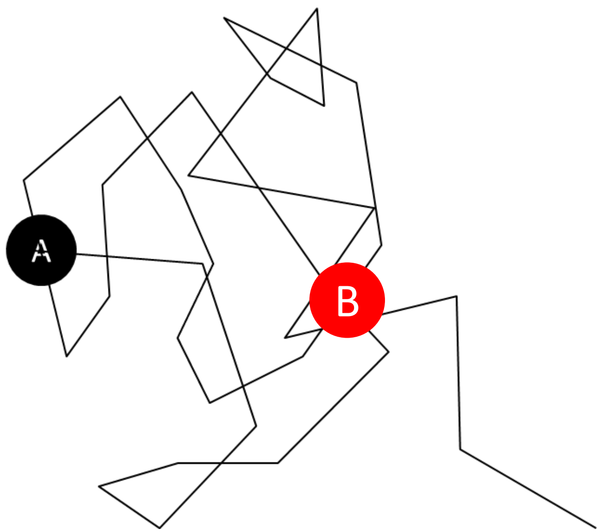
Lévy

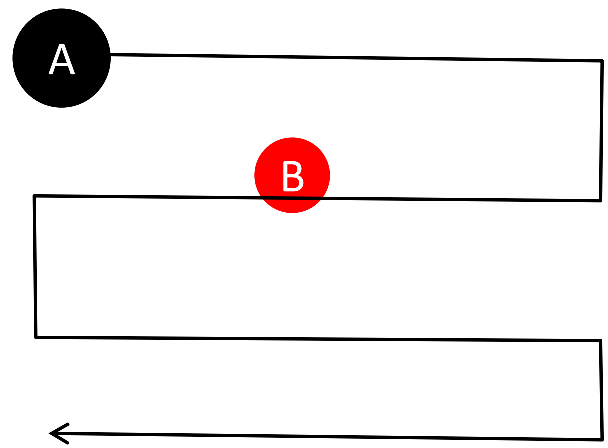


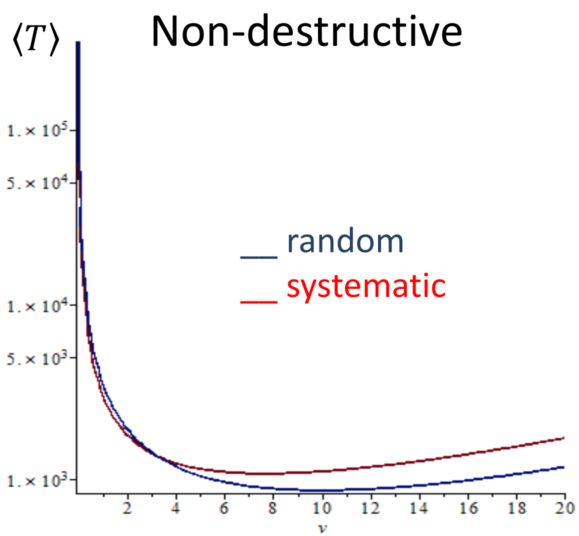
Exponential

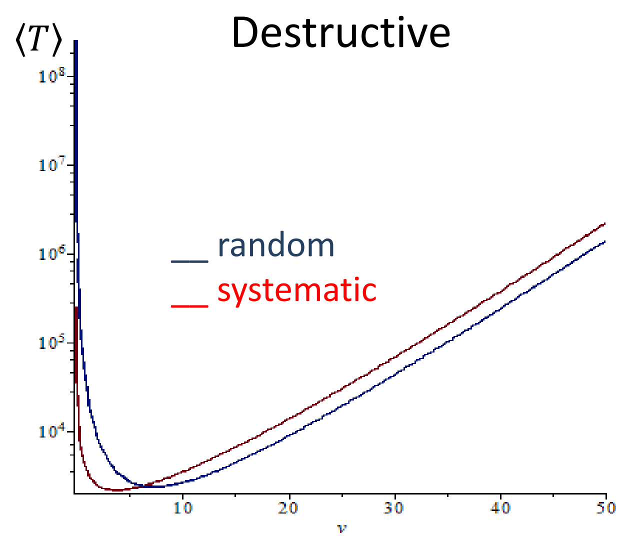


Random vs systematic strategies: MFPT



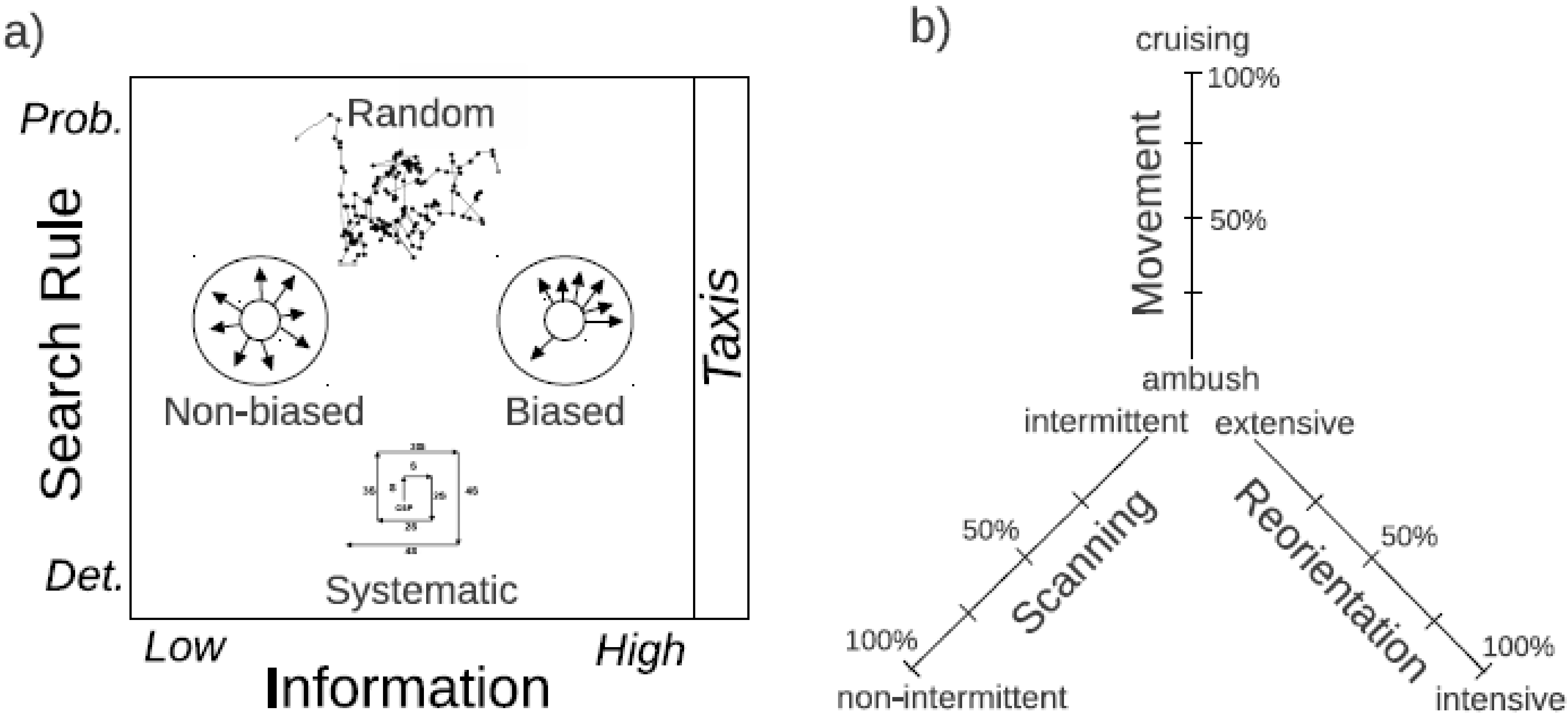




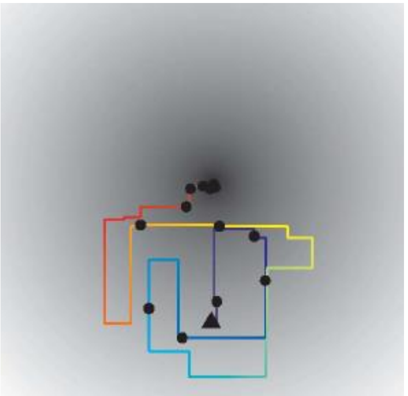


6. DISCUSSION: THE TARGET PROBLEM vs REAL SEARCHES

THE PLACE OF RANDOM SEARCHES IN REAL SCENARIOS



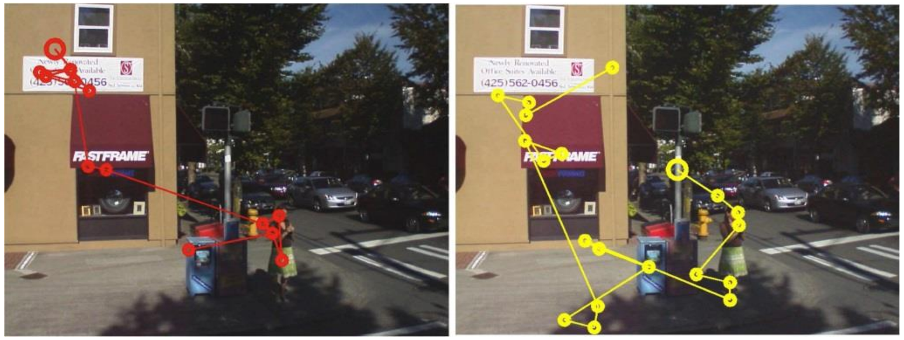
STRATEGIES TO DEAL WITH INFORMATION



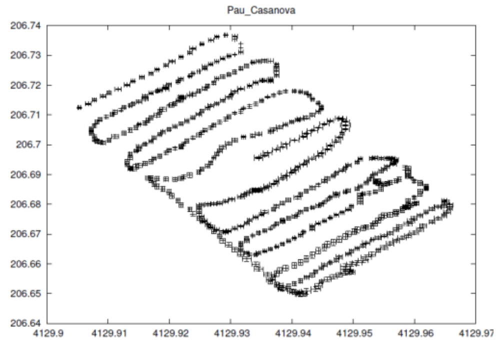
Infotaxis: search based on internal information maps

$$\overline{\Delta S}(\mathbf{r} \mapsto \mathbf{r}_j) = P_t(\mathbf{r}_j)[-S] + [1 - P_t(\mathbf{r}_j)][\rho_0(\mathbf{r}_j)\Delta S_0 + \rho_1(\mathbf{r}_j)\Delta S_1 + \ldots]$$

Value: search based on local information gathering



Heuristics: search based on *mental shortcuts*



EXPERIMENTAL EVIDENCES

Lévy Flights in Dobe Ju/'hoansi Foraging Patterns

Clifford T. Brown · Larry S. Liebovitch · Rachel Glendon

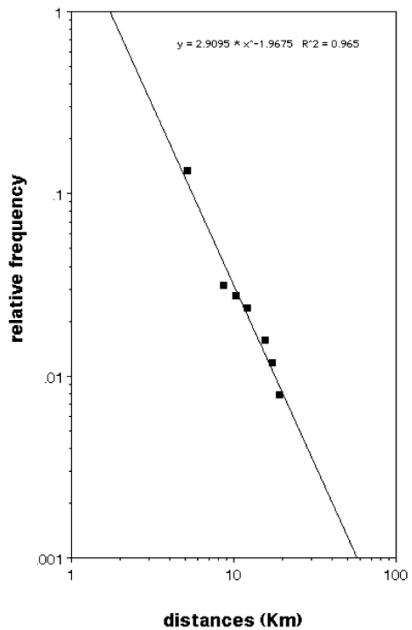
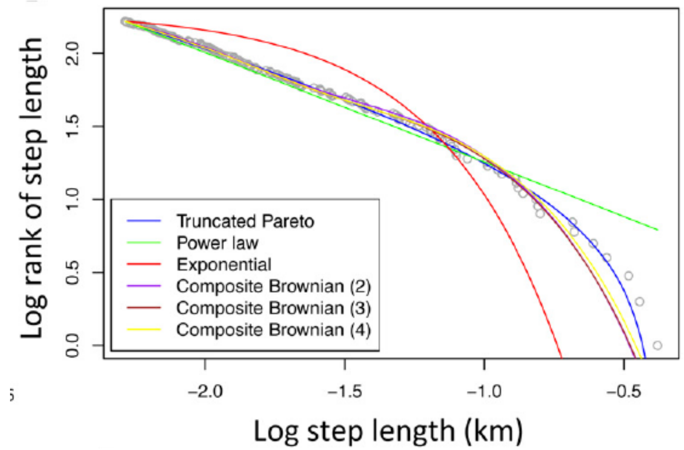


Fig. 1 Power law distribution of distances between campsites, exhibiting an exponent of 1.9675.

Evidence of Lévy walk foraging patterns in human hunter–gatherers

David A. Raichlen[a,1], Brian M. Wood[b], Adam D. Gordon[c], Audax Z. P. Mabulla[d,2], Frank W. Marlowe[e], and Herman Pontzer[f,g]



Lévy flights in human behavior and cognition

Andrea Baronchelli[a,*], Filippo Radicchi[b]

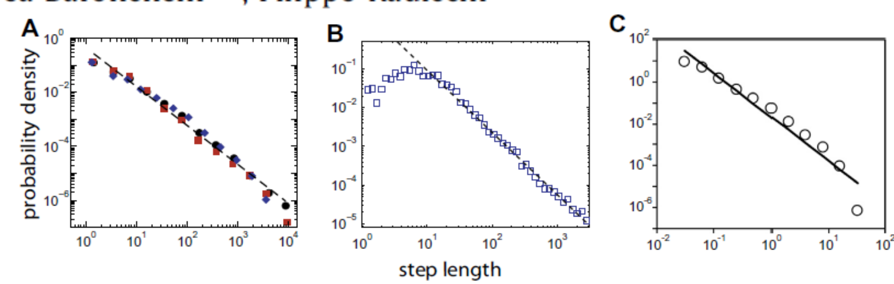
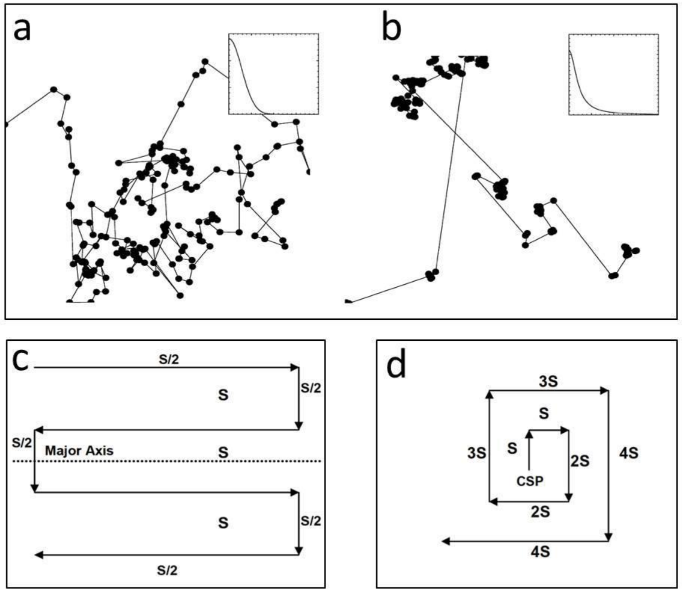


Fig. 1. Empirical evidence of the ubiquity of Lévy flights in natural systems. (A) Probability density of the difference between consecutive bid values in on-line auctions. Step lengths are measured in dollars. Adapted from Fig. 3A of Radicchi et al. [16]. (B) Probability density of the distance covered in human travels. Step lengths are measured in kilometers. Adapted from Fig. 1C of Brockman et al. [12]. (C) Probability density of the distance between different positions occupied by Atlantic cod (*Gadus morhua*). Step lengths are measured in meters. Adapted from Fig. 1D of Sims et al. [5].

6. DISCUSSION: THETARGET PROBLEM vs REAL SEARCHES (5/9)

SPECIFIC APPLICATIONS: SAR (standard protocols)

Search protocols (trajectories) recommended in SAR manuals can actually be seen as 1D search processes where the sweep pattern determines the detection probability.



For regular (horizontal or vertical) sweep in optimal conditions (sweep distance= $2r_d$) one has a path length of $l \approx L(L/2r_d)$ with ballistic movement. For this case one finds

$$\langle T \rangle = \frac{l e^{\gamma v} - x_0}{v} \qquad S(\infty) = 1 - \frac{v}{l\omega} \frac{e^{-\gamma v}\left(1 - e^{-\omega L/v}\right)}{1 - (1 - e^{-\gamma v})e^{-\omega L/v}}$$

…to be compared with the random case: $\langle T \rangle = \dfrac{L^2}{2D} g(x_0) + \dfrac{L^2 e^{\gamma v}}{2 r_d v}$

SPECIFIC APPLICATIONS: SAR (experiments)

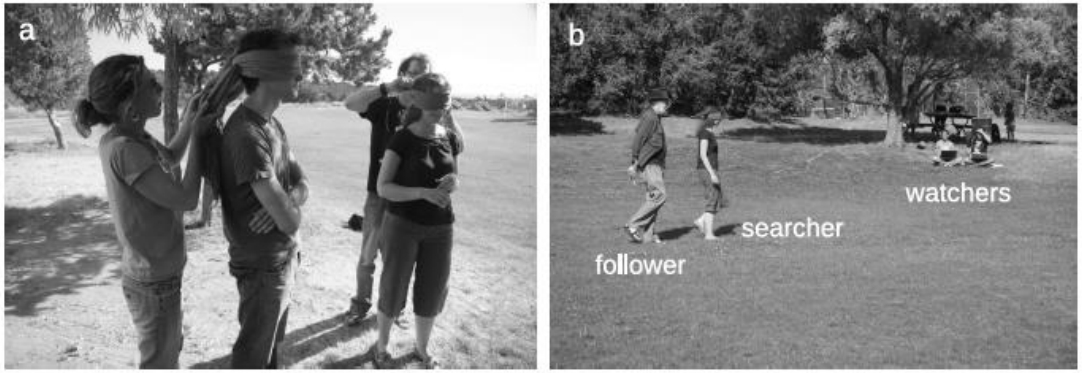
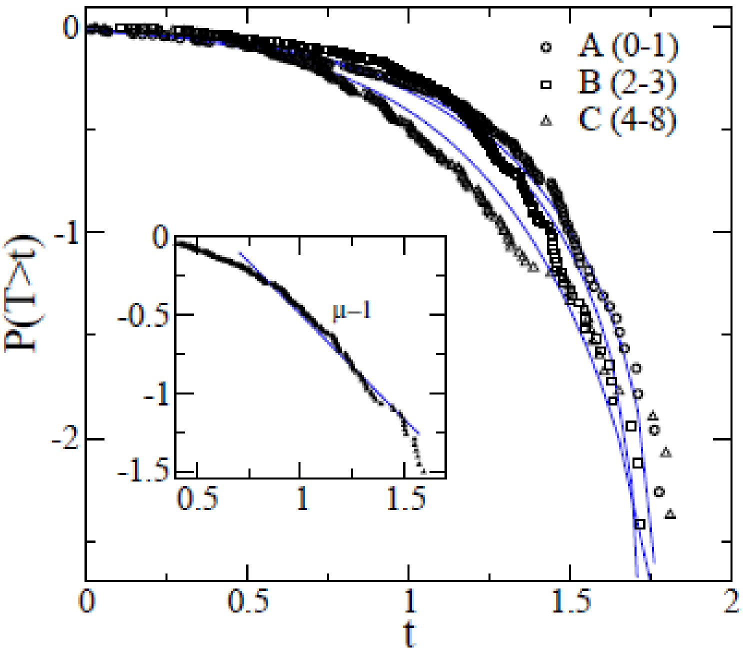
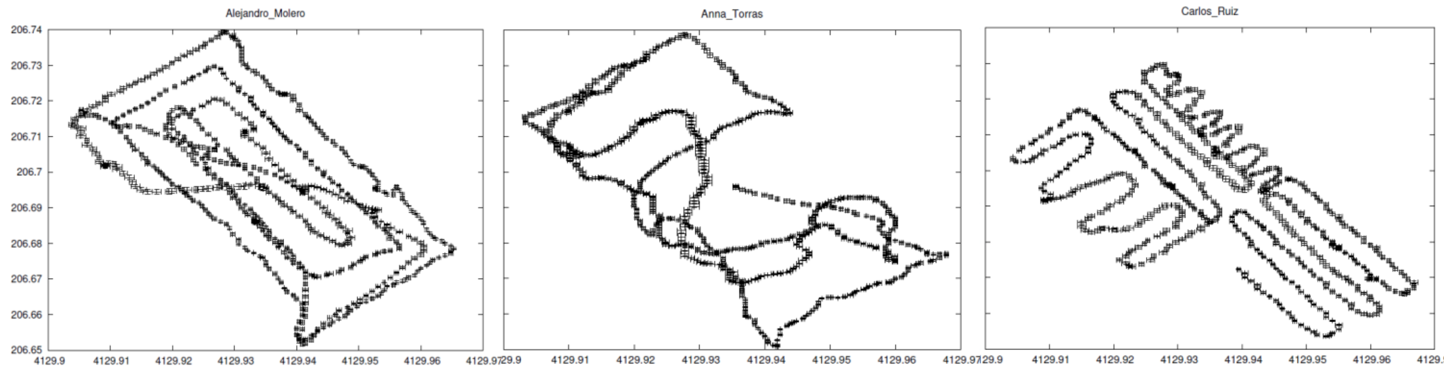
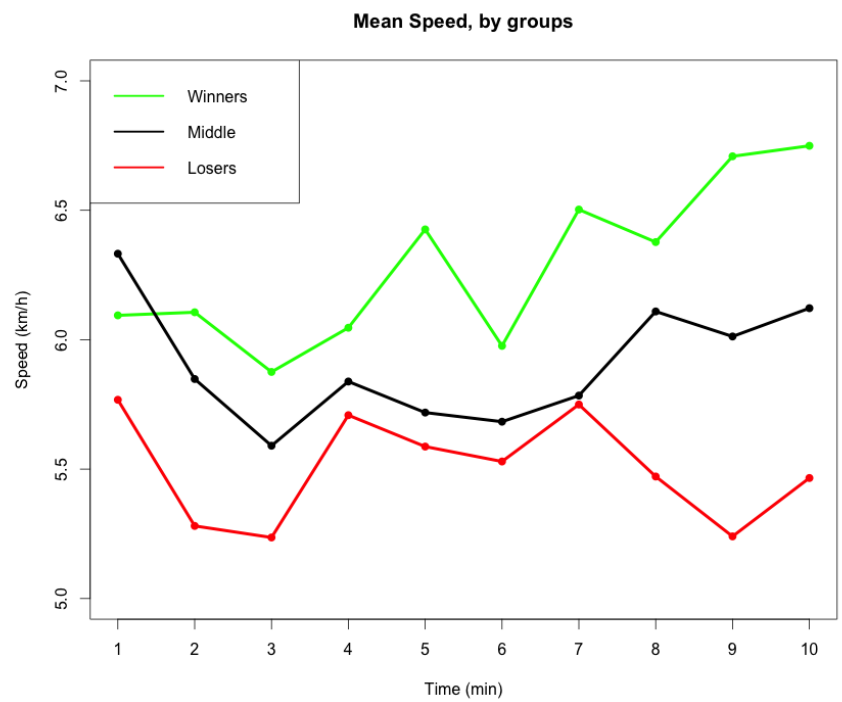


Fig. 9.4 19 blind folded volunteers searched the field during 10 min for 5 targets. a) Searchers were instructed to search for as many targets as possible with no prior knowledge of the theoretical background of the experiment nor what would be measured during the experiment. Each searcher was blind folded before entering the field, and was then lead to a random starting position in the field. b) During the experiment, the searchers remained blind folded while searching for targets, and were each escorted by a follower. During the search, the walking path of each searcher was observed by a watcher, who computed the length of time interevents between reorientations, using a software written for that purpose. Velocity was assumed to be constant. Photographs taken by Amelie Veron.

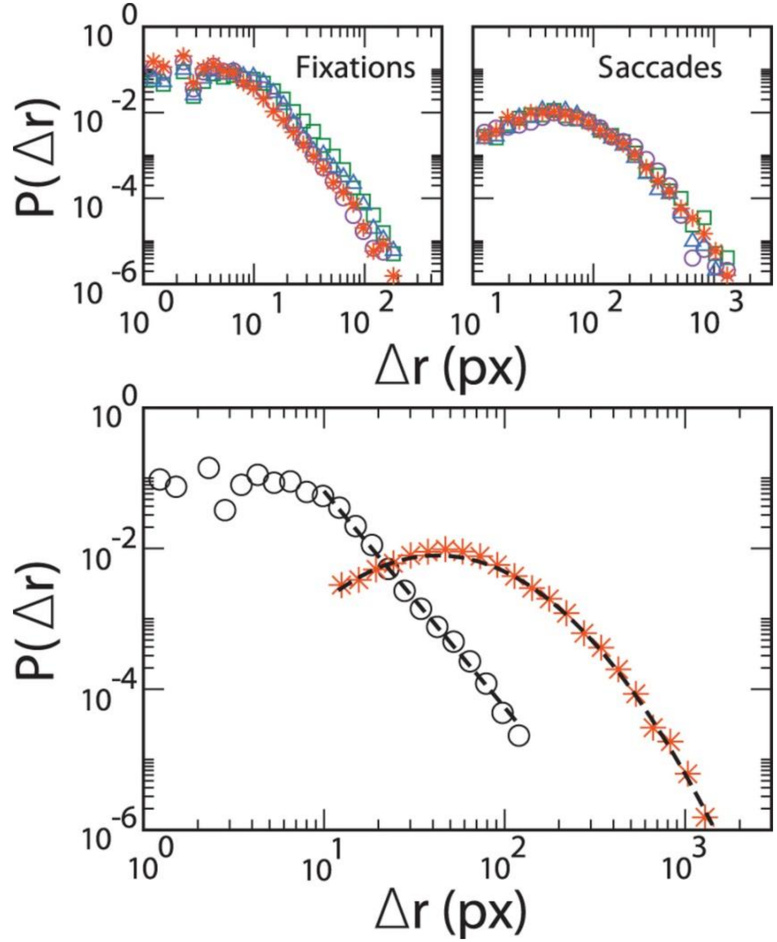


SPECIFIC APPLICATIONS: SAR (experiments)



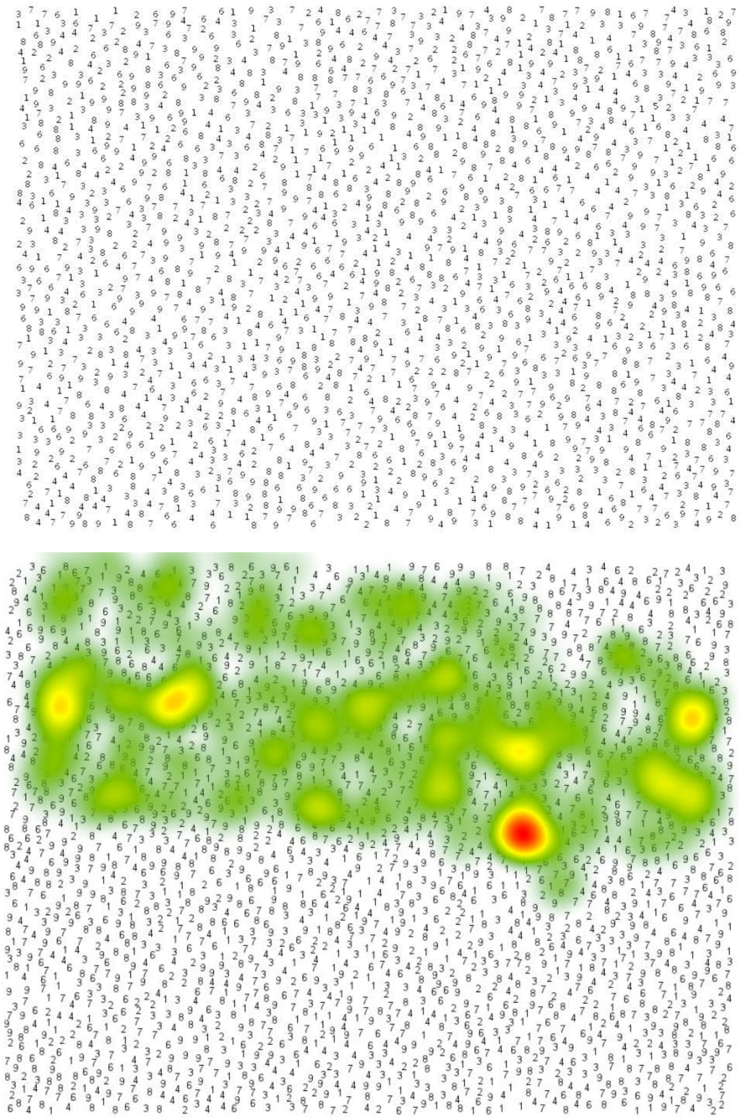


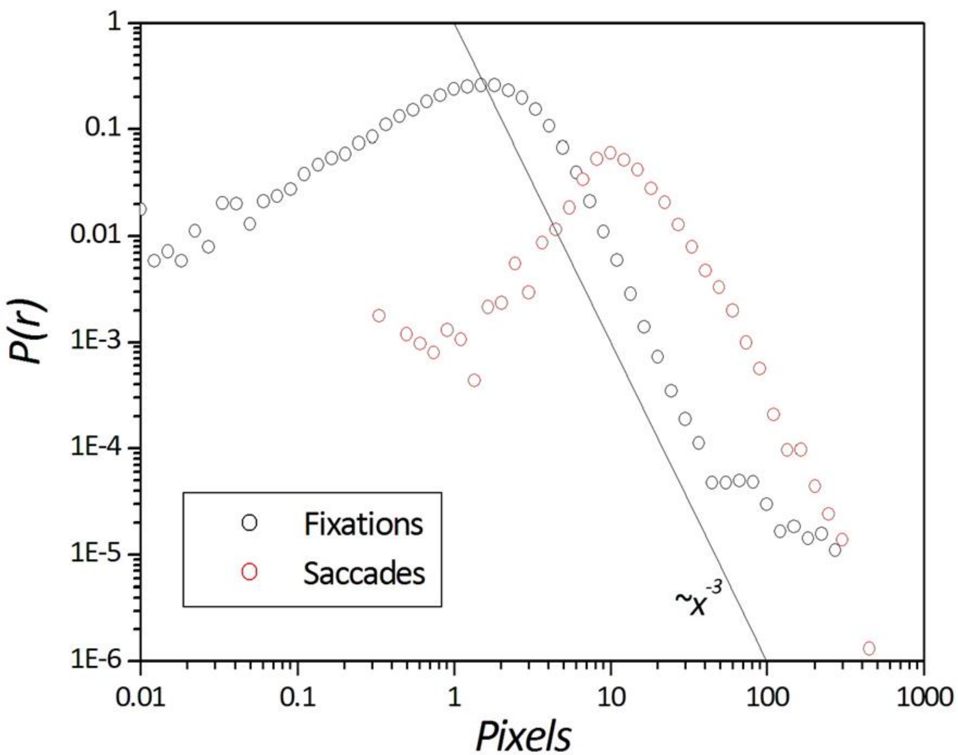
SPECIFIC APPLICATIONS: EYE-TRACKING



Credidio et. al. Sci. Rep. 2, (2012)

SPECIFIC APPLICATIONS: EYE-TRACKING





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