

Three Remarks on the Interpretation of Kant on Incongruent Counterparts

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In recent times, a number of authors have systematically criticized Kant's 1768 'proof' of the reality of absolute space. Peter Remnant may have been the first to do so, but many others have since joined him, either challenging the argument itself or showing how relationist conceptions of space can account for incongruent counterparts just as well as absolutist conceptions.¹ In fact, Kant himself abandoned his main conclusion soon after publication, favouring instead the doctrine of transcendental idealism. I do not see how the 1768 proof can be saved, nor will I defend it here.² However, in dismissing it some critics seem to have gone too far, and either failed to fully acknowledge Kant's contribution, or attributed to him thoughts he is unlikely to have had. Kant's treatment of incongruent counterparts in his *Dissertation* of 1770 has also met strong opposition. In particular, his claim that the difference between a pair of incongruent counterparts cannot be apprehended by means of concepts alone³ has been taken to be a mathematical falsehood. Indeed, incongruent counterparts have been shown to be mathematically distinguishable, with no intuitions needed for that purpose.⁴

This article addresses those criticisms. The first section interprets what Kant meant in his 1768 essay by claiming that the orientation of an incongruent counterpart has an 'inner ground'. It is not evident what he means there, and the relevant passages may seem equivocal. Section 2 interprets and defends Kant's apparently false claim (in the 1770 *Dissertation*) that the difference between incongruent counterparts cannot be apprehended by concepts alone. The article ends by briefly summarizing arguments by Arthur Prior, P. F. Strawson and Richard Gale on demonstrative identification and comparing them

with Kant's remarks on incongruent counterparts. This may help in the understanding of the Kantian claim discussed in section 2, and provide additional evidence for its truth. The analysis of orientation and form as having an 'inner ground', which Kant put forth in his 1768 essay as evidence for the reality of 'absolute space', was not completely rejected in later works but explained instead as grounded on a faculty of sensibility and no longer as a property of things independently of how they affect us. The 1770 claim that the difference between incongruent counterparts cannot be apprehended by concepts alone presupposes that the particular form of an incongruent counterpart remains the same regardless of the position and orientation of that object relative to other objects; hence it presupposes the analysis of form put forth in the 1768 essay, but explains its results differently. The remarks discussed in sections 1 and 2 below concern the same facts, and a better understanding of one will provide additional insight into the other; a similar effect is expected of section 3.

I. The Difference between Incongruent Counterparts Rests upon an 'Inner Ground'

Kant introduced the notion of an *incongruent counterpart* in his 1768 essay, describing it as 'a body which is exactly equal and similar to another, but cannot be enclosed in the same limits as that other' (2: 382). Clear examples are left and right human hands when minor differences in shape and size are disregarded. Even if they were perfect mirror images of each other, they would still not be enclosable within the same limits: left hands do not fit into right gloves and right hands do not fit into left gloves (without stretching, and thus deforming, the glove). Other examples include screws, human ears, snail shells and scalene spherical triangles.

Kant was not the first to notice the existence of incongruent counterparts,⁵ but he appears to have been the first to formulate a general method for constructing them:

let a body be taken consisting, not of two halves which are symmetrically arranged relatively to a single intersecting plane, but rather, say, a *human hand*. From all the points on its surface let perpendicular lines be extended to a plane surface set up opposite to it; and let these lines be extended the same distance behind the plane surface, as the points on the surface of the

hand are in front of it; the ends of the lines, thus extended, constitute, when connected together, the surface of a corporeal form. That form is the incongruent counterpart of the first. (2: 382)

Both the original hand and its counterpart have exactly the same characteristics except for orientation – that is, except for the fact that one of the hands is right, while the other is left – and location. Apart from that, they are *equal* (same magnitude) and *similar* (same in regards to the relations of their parts whenever each is considered in isolation, that is, without comparing it to anything else).

Kant showed that those objects were counterexamples to a proposition Leibniz and Wolff thought they could prove, namely that for any geometrical object, equality and similarity entail congruence (hereafter: ‘Leibniz’s thesis’).⁶ Although all counterparts are indeed similar and equal, some are incongruent. In fact, the counterparts of most familiar bodies that cannot be divided into two symmetrical halves – such as human hands and ears, screws, and snail shells – are incongruent.⁷ Those bodies cannot be enclosed within the same limits as their counterparts, no matter how you move and turn them around (without stretching and bending, that is, without changing their forms).⁸

Paul Rusnock and Rolf George have done a great deal to further a proper historical and philosophical understanding of Kant’s arguments. As they point out,⁹ ‘equality’ and ‘similarity’ have a long history in mathematics, which can be traced back to ancient Greece. Definitions of similarity for certain types of objects can be found, for example, in Euclid’s *Elements*.¹⁰ Leibniz, however, seems to have been the first to formulate a general concept of similarity, one not restricted to any particular type of object, but applicable to all. Ancient Greeks had a specific definition for each particular kind of object. Hence, they never could have entertained something like Leibniz’s thesis, which relies on a concept of similarity in general.

Leibniz classified mathematical concepts as either quantitative or qualitative: the former hold of objects in comparison to other objects, the latter of objects considered in isolation.¹¹ *Magnitude* would be a quantitative concept. We can say, for example, that a certain object is twice as large, half as long, etc. as another; but it makes no sense to say that an object is twice as large (or half as long) without comparing it to something else. ‘Big’, ‘small’, ‘long’, ‘short’ and the like are terms whose application requires comparison. Some objects, of course, provide

standards of measurement. Compared to a metre ruler, for example, one might say ‘this pen is 15 centimetres long’. In such cases a comparison is made nonetheless, the only difference being that one of the objects of comparison (the ruler) serves as a standard; being 15 centimetres is by no means a property an object has when regarded in isolation.

The *form* of an object, on the other hand, would be determinable without comparison: a sphere is a sphere independently of what it is compared to, and the same is true of any other geometrical form. Leibniz’s notion of similarity intended to capture sameness of form. Two objects are similar if they are indistinguishable when each is considered in isolation (without comparing it to anything else).¹² Pairs of spheres or cubes of different sizes, for example, cannot be distinguished when each member of the pair is considered in isolation; they are similar and have the same form. As Kant would point out, however, this is not a general rule: pairs of incongruent counterparts are also similar in that sense, but cannot be said to have the same form, as they cannot be enclosed in the same space.

As mentioned above, Leibniz and Wolff conceived equality and similarity as necessary and sufficient conditions for congruence: any two equal and similar bodies would *ipso facto* be congruent (enclosable in the same limits). Although we now know that to be false, we should first note that one could easily be inclined to think it true, especially if one accepts Leibniz’s definitions of *similarity* and *equality*. Those definitions intend to capture both sameness of form and sameness of magnitude, and it is natural to think that if two objects have the same form and the same magnitude, they should be geometrically indistinguishable (disregarding location, of course). If nothing, apart from location, distinguishes them, then they ought to be enclosable in the same limits (congruency).

However, there is more to the form of an object than Leibniz had thought. Incongruent counterparts are indeed similar, but are clearly – as one can perceive by looking at one’s right and left hands – different in form. They differ because the internal arrangements of their parts are diversely ordered, resulting in diverse orientations. But the orientation of an object is not discernible when that object is considered in isolation. Hence, despite having different orientations, a pair of incongruent counterparts is nonetheless *similar*. Since they are also *equal* (given the way they are constructed), according to Leibniz’s thesis they should be congruent, which they are not.

On a first approximation, orientation appears to be a quantitative Leibnizian characteristic, for it can only be discerned by comparison. It only makes sense to say of an arrow, for example, that it points up or down if one has in mind some point or system of reference relative to which up and down can be established. When each arrow is considered in isolation, nothing distinguishes this arrow: \rightarrow , from this one: \leftarrow ; nothing can be said of one that cannot also be said of the other. When compared to one another, or to something else – the page in which they are printed, for example – one immediately sees that they point in opposite directions, that they have different orientations. The same goes for objects such as a hand or a screw. Considered in isolation, there is nothing that can be said of either that cannot also be said of its counterpart. Both a right human hand and its counterpart have five fingers, an index finger between the middle finger and the thumb, and so forth. When each is considered in isolation, they are perfectly alike; hence, in this case too, orientation would count as a quantitative Leibnizian characteristic. But whereas it is unproblematic to regard two arrows that point in different directions as *similar* and as having the same form, a human hand and its counterpart do not have the same form (for they are not enclosable in the same limits) and thus should not count as similar, if similarity is sameness of form. Hence, Leibniz's concept of similarity, which purported to capture sameness of form for any two objects whatsoever, fails to do so.

With his 1768 essay Kant showed that orientation is a component of the form of at least some objects, including hands and screws. Leibniz's analysis of geometrical form was lacking, and his corresponding definition of similarity was incomplete. Yet it is not immediately evident how to analyse further the concepts of form and similarity so as to account for orientation. At first sight, as we have just seen, orientation seems to be a quantitative characteristic, more akin to magnitude than to form, since it requires comparison to be applied. However, since differences in the orientation of some objects entail differences in their forms, it also seems reasonable to think of orientation as properly pertaining to the form of an object, hence as a qualitative characteristic. This seems to lead to a dilemma about whether to take orientation as a quantitative or a qualitative notion.

Adding to the dilemma, objects such as human hands – though not arrows – appear to have a particular orientation on their own. They clearly do not change their oriented form merely by being moved or turned around. The overall form of those bodies is determined by their

orientation, since differences in orientation entail non-enclosability in the same limits. One could therefore expect orientation to be a component of the form of an object, something an object would have even when considered in isolation. On this account, it should count as a qualitative characteristic.

Instead of deciding whether to view orientation as quantitative or qualitative, Kant seems to have taken an alternative route by coming up with a distinction of his own. In his 1768 essay, he does not mention Leibniz's qualitative/quantitative distinction, but speaks instead of the difference between incongruent counterparts in terms of an 'inner' ground:

Since the surface which limits the physical space of the one body cannot serve as a boundary to limit the other, no matter how that surface be twisted and turned, it follows that the difference must be one which rests upon an inner ground. This inner ground cannot, however, depend on the difference of the manner in which the parts of the body are combined with each other.¹³

It is tempting to associate Leibniz's quantitative/qualitative distinction with a Kantian inner/outer ground distinction, but we are better off avoiding such association, for it brings an equivocation to the Kantian text.¹⁴ Instead we can read 'inner ground' as Kant explicitly describes it: as applicable to those characteristics that remain unchanged 'no matter how that surface be twisted and turned', that is, no matter how the object is moved and turned around without being deformed. Then the orientation or direction towards which an arrow points has an *outer* ground, for it *does* change when moved and turned around. Hands, screws and snail shells, on the contrary, are unlike arrows in that they not only have a changing orientation relative to other objects (the direction towards which, say, the palm of my hand faces) but also an oriented form (right or left), which *does not* change by moving or turning them around. That is perhaps what led Kant to claim that the orientation of a hand – that which sets a pair of incongruent counterparts apart – has an 'inner ground'. The orientation and form of an object is then independent of its relations to other objects, and should remain unchanged even when that object is regarded in isolation, albeit discernible only by comparison. Also the magnitude of an object would seem to have an inner ground, but Kant says nothing about this in his 1768 essay. In any case, magnitudes do not change by

merely moving or turning an object around, and thus seem not to depend on comparison, even if they can only be discerned or apprehended by comparison.

Immediately after introducing the notion of an 'inner ground', Kant put it to use in his well-known 'lone hand' argument. This is a much-contested argument, but perhaps for the wrong reasons. Kant invites the reader to 'imagine that the first created thing was a human hand' (2: 382). Now, if it is true that the orientation and hence the form of a hand has an inner ground, it should be expected that; 'the action of the creative cause in producing the one would have of necessity to be different from the action of the creative cause producing the counterpart' (ibid.). The same idea underlies the rather lengthy discussion, with which the essay begins, of examples of things we recognize by their oriented form (snails, beans, hops, etc.). In those natural species the oriented form is inherited and does not depend on spatial relations to other things: 'the cause of the curvature in the case of the natural phenomena just mentioned is to be found in the seeds themselves'.¹⁵ Kant contrasts those natural phenomena that are invariant with respect to location and movement with the opposite directions in which winds change in the northern and southern hemispheres. Those changes, Kant implies, do not have an inner ground, for they are affected by the directions of the movements of the sun and the moon.¹⁶

That a certain characteristic has an inner ground is not an epistemological claim having to do with how we recognize or know it, but a metaphysical claim: characteristics that have an inner ground pertain to the things themselves regardless of their relations to other things, whereas characteristics that have an outer ground are derived from or caused by those relations. The form of an object, as we have just seen, would then have an inner ground, for it does not change by merely altering those relations with moving or turning the object around. Since orientation was found to be a component of the form of some bodies (as the examples of incongruent counterparts showed), it follows that it, too, has an inner ground in those bodies. That is the case even though, as Kant acknowledges, we can only perceive the orientation, and hence the form, of an object by comparison.¹⁷

The fact that there are spatial properties to be found in the objects themselves regardless of their relations to other things was taken by Kant in 1768 to indicate a general flaw in Leibniz's relationist conception of space. Later Kant would explain the objectivity of those

properties as grounded in the forms of our sensibility. Before moving on to those later developments, let us dwell a little on what Kant thought (in 1768) were the implications of the fact that there are such properties.

Since orientation has an inner ground, it follows – Kant argues – that when we imagine that the first created thing was a human hand, that hand must have a particular form and hence a particular orientation, right or left. Notice that Kant does not say that we would be able to recognize it as right or as left, since we can only do so by comparison, and in the case of the lone hand there would be nothing to which it could be compared. However, Kant does claim that it would have to be either right or left nonetheless. If it is a hand, it must have the form of a hand, and therefore it must have one of two possible incongruent forms, that of a right hand or that of a left hand. The argument relies on the view that there is no such a thing as a hand that is neither right nor left, which is derived from the claim that form and orientation have inner grounds. This is not a gratuitous assumption. But it is somewhat obscured in discussions that focus solely on the sentences immediately surrounding the lone hand passage. There Kant claims that if the lone hand were neither right nor left, then it ‘would fit equally well either side of the human body; but that is impossible’.¹⁸ It cannot fit both sides of a human body because it has a definite form, and the hands that fit each side of a human body are incongruent (that is, they have different forms).

The claim that even a lone hand must be either right or left is supposed to show that Leibniz’s relationist conception of space is wrong. According to Leibniz, the form of any object is given by the relations that the parts of that object have among themselves. Since, as Kant showed, those relations are the same in a right and in a left hand when each is considered in isolation, it follows that if the first thing created had been a lone hand, then on Leibniz’s account it would be neither left nor right. As we have just seen, Kant takes that to be obviously false, for it would amount to saying that the hand would have an amorphous form, which ‘would fit equally well either side of the human body’.

Kant then infers that absolute space must be real, for ‘the determinations of space are not consequences of the positions of the parts of matter relative to each other. On the contrary, the latter are the consequences of the former’ (2: 383). To this inference, however, the relationist may reasonably object: if the first thing created had been

what we now call a right hand and the rest of the universe had been created as a mirror image of our actual universe, would we still say that the original hand was a right hand? Instead of fitting the side of the body opposite to the one where most people's hearts are (what we usually call the 'right side'), it would fit the other side. Hence, a relationist can reasonably say that, although the lone hand would have a form, we cannot say that it is either right or left. Attributions of 'rightness' or 'leftness' require comparison, and in the lone hand case there is nothing to which the hand can be compared. We cannot say which side of a handless human body it would fit either, for such body does not exist in the lone hand universe and we do not know whether that body would be similar to the ones we have or to the counterparts of the ones we have. It is only after it is introduced that we can establish the side to which the hand will fit.

Kant's argument, however, does not need the stronger claim that the lone hand is either right or left, but only the weaker claim that it has a form. If it has a definite form – and it should have, if it is to count as a hand at all – then it will fit one side of the body but not the other. This is a plausible claim. The problem with Kant's 'proof' of the reality of absolute space lies in not in the lone hand thought-experiment, as has been suggested, but in the inference Kant draws from it, purporting to prove the reality of absolute space. As I have indicated, there is no evident reason to think a relationist might not be able to account for orientation and incongruence just as elegantly as an absolutist.¹⁹ Kant's arguments here simply do not settle the issue.

Despite the problems with Kant's inferences from the lone hand experiment to the reality of absolute space, the analysis of the form of an incongruent counterpart as having an 'inner ground' is not as implausible as it has been thought to be. Contemporary commentators tend to emphasize the fact that orientation and form cannot be 'intrinsic' since any object may change its orientation if moved through a non-oriented space (such as Möbius Strip or a Klein Bottle) or flipped around in a fourth (or higher) dimension.²⁰ Although this is true, neither Kant nor his contemporaries regarded such mathematical possibilities as physically entertainable. Yet Kant's remarks on 'inner ground' need not be interpreted as incompatible with such possibilities. All that is implied by his remarks is that objects such as human hands have, at any given moment, an oriented form that does not depend on its relations to other things. Moreover, the fact that the form and orientation of a hand depend on the topology and geometry of space speaks in

favour of the ‘reality’ of space, not against it. Hence, it seems not only consistent with Kant’s account but also to provide additional – albeit only insufficient – evidence for it.

2. Differences between Incongruent Counterparts cannot be Apprehended Conceptually

Although Kant fundamentally rethought many of his ideas in the late 1760s and beyond, he kept returning to those he had formulated in the 1768 essay – even well into the Critical period. He argued in the 1770 *Dissertation* that our ability to apprehend incongruent counterparts shows we have a faculty of sensibility distinct from the intellect. As in the 1768 essay, the 1770 argument relies heavily on the fact that there are differences among the forms of oriented objects, such as pairs of hands that remain invariant no matter how those objects are spatially related to other objects. The spatial properties which were ascribed an ‘inner ground’ in 1768 will be, from 1770 onwards, thought to be grounded in the forms of our sensibility. There is clearly a shifting of gears from 1768 to 1770, and it is not immediately evident how the same facts that were thought to entail the existence of absolute space can later be thought to bear evidence for transcendental idealism. In particular, it is not evident why we should need intuitions to apprehend the form of an incongruent counterpart.

In §15C of the 1770 *Dissertation*, Kant states his thesis: ‘Which things in a given space lie in one direction and which things incline in the opposite direction cannot be described discursively nor reduced to characteristic marks of the understanding by any astuteness of the mind’.²¹ This has been thought to be a mistake, and the claim that ‘in these cases the difference, namely, the incongruity, can only be apprehended by a certain pure intuition’,²² has been interpreted as saying that a conceptual *distinction* between incongruent counterparts is impossible, which is a falsehood. On this reading, Kant would be arguing that differences in orientation can only be apprehended sensibly, for if we rely merely on conceptual descriptions, then the orientation and form of, say, a right hand becomes indistinguishable from that of its left counterpart. If taken literally, this is indeed false – as several commentators have pointed out.²³

It is mathematically possible to *distinguish* incongruent counterparts by means of concepts alone; intuitions are unnecessary for that

purpose. A conceptual distinction between incongruent counterparts can easily be drawn, for example, with the concepts of similarity, equality and *orientation*: two counterparts are congruent iff they are similar, equal *and* of the same orientation. All that is therefore needed is the introduction of a concept of orientation. But if things are so simple, why did Kant think otherwise? Felix Mühlhölzer maintains that it was because he lacked the necessary technical tools. These would only have been developed in the following century:

Neither the deductive nor the conceptual resources of modern logic and mathematics were available to him. He saw clearly that *his* logic could not do justice to mathematical (and especially geometrical) knowledge: neither to the conclusions drawn in Euclidean geometry, nor to the concepts required for a complete grasp of the facts of geometry. Kant used intuition to fill these gaps, a move entirely justified at that time.²⁴

Here is a simplified version of Mühlhölzer's method for distinguishing incongruent counterparts without the aid of intuitions: consider a three-dimensional space, determined by three Cartesian axes V, H, F , where V stands for the vertical axis, H for the horizontal, and F for the axis perpendicular to both. The form of any object in that space can then be described as a set of points in that space. Each point is identified by an ordered triple $\langle x, y, z \rangle$, where each element of the triple corresponds to a certain point in one of the three axes. The form of a human hand can thus be described by a set of ordered triples $\{\langle x_1, y_1, z_1 \rangle, \langle x_2, y_2, z_2 \rangle, \dots, \langle x_n, y_n, z_n \rangle, \dots\}$. Given Kant's method for constructing counterparts presented above (nowadays known as reflection in a plane), we might as well suppose – so as to make our reasoning easier – that the plane of reflection coincides with one of the axes, say, the V axis, and is perpendicular to the other two. Then any point $\langle x, y, z \rangle$ when reflected becomes $\langle x, -y, z \rangle$. If a human hand is reflected, its counterpart will be another human hand, but of opposite orientation, and their diverse orientations will indeed be noticeable in their respective descriptions: one of them would be a set of ordered triples of the form $\langle x_i, y_i, z_i \rangle$, whereas the other would be described by a set of ordered triples of the form $\langle x_i, -y_i, z_i \rangle$. This method suffices to *distinguish* any pair of incongruent counterparts, for we would then have a different description for each. No intuition is needed here. Apparently, this would also show the falsity of Kant's 1770 statement, mentioned earlier.²⁵

Paul Rusnock and Rolf George²⁶ agree that Kant's appeal to intuitions was ill grounded but disagree with Mühlhölzer's claim that Kant was justified in bringing them in to fill a gap in the mathematics of his time. They say that Kant already had all the technical means needed to draw a conceptual distinction. Indeed, 'the difference between similar and equal things which are not congruent' (2: 403) is a precise description of the difference. But they concede that this would amount to saying that incongruent counterparts are different because they are non-identical, which would hardly count as a description of the difference, and even less as a clarification. Since one of the aims of philosophy for Kant was clarification,²⁷ he would obviously be unhappy with such an approach, as Rusnock and George themselves acknowledge.

Kant did introduce a term for the characteristic that sets pairs of incongruent counterparts apart: direction (*Gegend*).²⁸ But he never considered this as a properly discursive concept: it is 'a concept that can be constructed, but in no way can be made clear as a concept in the discursive mode of cognition'.²⁹ Despite Kant's indications, Rusnock and George have difficulties understanding Kant's reluctance to acknowledge the possibility of distinguishing incongruent counterparts without intuitions. They think an explanation might be sought in an alleged conceptual conservatism in mathematical matters, which would have been due to a belief in the completeness of existing mathematics, parallel to his belief in the completeness of logic.³⁰ Since mathematical methods would be above all suspicion, and since the concepts of mathematics would have achieved the same degree of completeness Kant attributed to logic,³¹ the 'paradox' of incongruent counterparts would have its origins in philosophy:

His recognition of incongruent counterparts, on this view, would not be understood by him as the discovery of the need for conceptual changes in the geometrical systems he knew, but rather of something beyond the reach of all concepts. Thus, instead of pointing out an error in Euclid and Leibniz, and looking to mathematics for a solution, Kant decided that the difficulty was rooted in a philosophical assumption. Put another way, Kant had too much respect for the mathematical acumen of others, and not enough for his own.³²

A more sympathetic reading of Kant's writings on the issue is in order here. Note that it is by no means evident what Kant meant by 'apprehended' – 'notari', in the original Latin text – in the passage

quoted earlier. The authors I have been discussing take Kant to be claiming that differences among incongruent counterparts cannot be *distinguished* conceptually. If this were the only or the best possible interpretation, then, as we have just seen, Kant's statement would indeed be false, and an explanation for his mistake would have to be sought.

There are, however, several problems with this line of interpretation. The first is that Kant's aim in all his published discussions of incongruent counterparts was to make a philosophical and not a mathematical point. Yet, as pointed out above, the conceptual distinctions presented by Mühlhölzer, Rusnock and George, and others, can at best be regarded as contributions to mathematics. From a philosophical perspective, they hardly clarify anything, and amount to stating verbally a difference we are already aware of, without explaining how one can become aware of such differences in the first place. This is at odds with the clarity that philosophy should bring about, and with the diverse aims and methods which Kant persistently assigned to mathematics and philosophy at least since 1763, when he submitted his essay on method to the Berlin Academy.³³

A second problem with that line of interpretation is that it conflicts with the opening third of the 1768 essay, in which Kant systematically discusses ordinary visual patterns – the outer shape of natural species such as beans, hops, snails, etc., a handwritten page, the position of the stars in the sky, etc. – that we recognize and identify partly because of the orientation in which they are disposed. The purpose of those examples is twofold: they show that form and orientation are characteristics that remain invariant no matter how those objects are moved or turned around, and they also show that we are capable of singling out oriented forms. Kant points out that 'the difference in the directions is so important and so closely connected with the impression made by the visual object that the self-same writing, when viewed with everything transposed from right to left, ceases to be recognizable' (2: 379). It is recognition of directions that is crucially at stake also in navigation:

the most precise map of the heavens, if it did not, in addition to specifying the position of the stars relative to each other, also specify the direction by reference to the position of the chart relative to my hands, would not enable me, no matter how precisely I had it in mind, to infer from a known direction, for example, the north, on which side of the horizon I ought to expect

the sun to rise. The same thing holds of geographical and, indeed, of our most ordinary knowledge of the position of places.³⁴

The information provided by a map is in itself useless if one does not know how to relate the map to our bodies, that is, to the place we are at now and the direction we are facing. Such additional information cannot be rendered in terms of the way things are ordered in the map. This is something that mere conceptual distinctions cannot establish. What is needed is an information of the sort ‘you are now here’ (pointing to a place in the map), ‘and your map should face that way’ (indicating how one should hold the map). Concepts (understood as Kant understands them, that is, as universal representations)³⁵ certainly play a role in the identification and recognition of places, directions, spatial patterns and forms, but, as we will see below, they cannot suffice.

A third, connected, problem with that line of interpretation is that it also conflicts with the use Kant made of his examples of incongruent counterparts in the 1770 *Dissertation*, where he argues that we have not only a faculty of concepts, but also a faculty of intuitions. Intuitions are not presented there as a supplement to the limitations of our intellect, but as providing cognitions of a wholly different sort: concepts are universal, while intuitions are singular representations. Accordingly, incongruent counterparts are called upon as examples of things whose unique form we would not be able to identify if our cognitive capacities were purely conceptual. Pairs of incongruent counterparts provide a particularly compelling example for Kant’s claim, for there is a perceivable difference between them. Those differences, however, are of a purely spatial nature, and – Kant argues – cannot be ‘compounded from sensations’ nor ‘derived from some universal concept of space’, but ‘can only be *apprehended* concretely, in space itself’ (2: 402–3). To understand what ‘right’ and ‘left’ mean, one must already have an intuition of spatial forms: spatial relations and spatial differences can only be represented as relations within a single unique space.³⁶ We are constrained, in imagination and perception, to represent a human hand as either right or left, but this is not something imposed on us by the way we think of hands. Rather this is something to which we are constrained given the way hands are given to us. It is this uniqueness of the particular form of a hand that cannot be ‘apprehended’ by universal representations (concepts).

To say that with concepts alone we can *distinguish* incongruent counterparts is not something Kant would have to deny. It seems that by

‘apprehended’ Kant did not mean – as the above-mentioned authors suggest – ‘distinguished’. Rather, it seems that his point was that concepts are not sufficient for the identification and recognition of particular forms. On this more charitable interpretation, Kant’s claim not only agrees with his other writings mentioned above, but also turns out to be true. Conceptual distinctions can be understood as the outcome of a capacity to form classes of objects according to a rule, such that anything that satisfies the rule belongs to the specified class. But the identification of particular spatial forms requires an additional cognitive capacity. Consider, for example, the class of human hands. It can be easily distinguished from the class of human knees. There are certain characteristic marks which set them apart (for example, hands have fingers, knees do not), and therefore at least some conceptual descriptions true of one will be false of the other. The identification of the particular form of a hand or a knee, however, is a capacity that cannot be reduced to the formation of classes. Conceptual distinctions yield only additional, more specific (less comprehensive) concepts, not particular spatial forms or things.³⁷

Suppose we distinguish a pair of diversely oriented hands using something like Mühlhölzer’s technique. One hand will be described by a set of ordered triples of the form $\{ \langle x_1, y_1, z_1 \rangle, \langle x_2, y_2, z_2 \rangle, \dots, \langle x_n, y_n, z_n \rangle, \dots \}$ whereas the other will be described by a set of ordered triples of the form $\{ \langle x_1, -y_1, z_1 \rangle, \langle x_2, -y_2, z_2 \rangle, \dots, \langle x_n, -y_n, z_n \rangle, \dots \}$. Suppose, additionally, that we name the former ‘right’ and the latter ‘left’. One could then take that pair of hands as a standard, and by comparing any other hand to those two, be able to classify them as either left or right. Kant’s point was that merely being able to do that would still not allow us to identify the particular form of any given hand; an additional, non-conceptual capacity is still needed for that. Making the description more precise will not do. It would if the differences between the hands were not purely spatial, but had to do with, say, their colour or strength. However, since they are identical in all respects except orientation, the matching up of a hand with a labelled standard presupposes the identification of the form of the things that are being matched up. A set of points of the form $\{ \langle x_1, y_1, z_1 \rangle, \langle x_2, y_2, z_2 \rangle, \dots, \langle x_n, y_n, z_n \rangle, \dots \}$ does not indicate in any way whatever the actual form of the hand described. It is indeed different from the hand described by the set $\{ \langle x_1, -y_1, z_1 \rangle, \langle x_2, -y_2, z_2 \rangle, \dots, \langle x_n, -y_n, z_n \rangle, \dots \}$, but nothing in either description tells us the form of either hand. If we are given the two descriptions, we can construct the first as

a left hand and the second as a right hand, or the other way around. Descriptions of this sort allow us to tell incongruent counterparts apart, but do not tell us which is which (which is the right-handed and which is the left-handed counterpart).³⁸

Of course, we can arbitrarily label either one as 'right', and the other as 'left'. But arbitrary labelling does not resolve the issue. The matter is not linguistic. If all we had were purely conceptual capacities plus labels, we would still be unable to identify and recognize the form of any given hand. If we were given a description, we would then be able to match it up with one of the labels, but we would remain unable to grasp the form of the hand described.³⁹

Moreover, it is only once the form of a given hand has been identified that we can judge (relative to a certain standard) whether it deserves the label 'right' or 'left'. Intuitions are therefore needed not only to match up cases (that is, a standard right hand with other instances of right hands), but also, and more importantly, to allow for the identification of the form of the things matched up. Without such a non-conceptual cognitive capacity, even if we were still able to draw distinctions, we would neither be able to identify the form of an individual object nor to judge which label it deserves (given a certain standard). The recognition of an object as an instance of an orientation class (say, right or left) requires that it be possible to identify the form of that object prior to judging it thus or so. This is a requirement that has to be satisfied in order for judgements to be objective.

This, I think, suffices to make the case for a more charitable reading of Kant's claim. Additional corroboration may also be found in Kant's uses of incongruent counterparts in his writings of the critical period. Incongruent counterparts are absent from the *Critique of Pure Reason*, but do appear in the *Prolegomena* (§13)⁴⁰ and again in the *Metaphysische Anfangsgründe der Naturwissenschaft*.⁴¹ In these texts incongruent counterparts figure as evidence for the transcendental ideality of space. The point there is that if we were capable of knowing things as they are in themselves, then the understanding alone should suffice to know them (the manner in which they relate to our sensibility should be irrelevant). However, concepts alone afford us no means of identifying the form of oriented objects. Since we are clearly able to identify the forms of such objects, we must have a non-conceptual faculty of cognition. Hence, knowledge of spatial forms rests at least in part on the way they affect us and cannot be knowledge of things in themselves. In these later uses of incongruent counterparts, the basis for

Kant's new claim (that space is transcendently ideal) is the same as before: conceptual capacities alone afford no knowledge of the specific form of an incongruent counterpart. Those forms are given to us: we cannot but perceive and imagine objects such as hands as having a particular oriented form. Hence, 'apprehension' of their forms is not grounded on the way we classify them, but rather on the way we are constrained to perceive them and relate them to our bodies.

3. Incongruent Counterparts and Demonstrative Identification

In the contemporary analytic literature, the Kantian claim that conceptual capacities alone cannot explain all our cognitive capacities is oftentimes put forth in claims about demonstrative identification, in particular, in the claim that non-demonstrative modes of identification cannot fulfil the roles played in our language by 'demonstratives' (or 'indexicals').⁴² In what follows, some of the arguments supporting that claim are briefly reviewed, as they shed additional light onto the Kantian remarks we have been discussing and corroborate their truth.

In a demonstrative identification,⁴³ the thing identified is neither singled out by its properties nor by its proper name (if it has one), but by its spatio-temporal relation to the subject of the identification. An ordinary utterance of 'that book' by someone who points to something resembling a book is a typical example, even when the thing being pointed to is not really a book but merely resembles one.⁴⁴ Similarly, for most uses of 'right', 'left', 'up', 'I', 'here' and 'now': the things identified are singled out primarily by their spatio-temporal relations to the subjects of the identifications, not by their proper names (if they have proper names) nor by the descriptions associated with them.⁴⁵ In contemporary jargon, Kant's claim amounts to saying that without relying on some demonstrative identification we cannot identify or recognize the form of an oriented object.

Here is a clear and persuasive way the general point about demonstrative identification has been made. Suppose someone says, pointing in a certain direction, 'the library is *over there*'. There are two basic ways of trying to convey that information without using demonstrative identification.⁴⁶ The first is to replace 'over there' with the name of a place, such as an address or a spatial coordinate, for example, '425 South Morgan Street'. The second is to relate it to something whose

position is already known, for example, ‘100 metres south of University Hall’. Both these attempts fail, however. The original utterance allowed for the identification of the place where the library is, whereas the modified versions do so only if we either already know where South Morgan Street is or where University Hall is. If we do not – and of each thing that there is, we are either now ignorant or were at some point in the past – then demonstrative identification is again needed: ‘the library is at 425 South Morgan Street, and *that* is South Morgan Street’, ‘the library is 100 metres south of University Hall, and *that* is University Hall’. One could try to eliminate the demonstrative element from the second half of each of those sentences, but the same reasoning would once again apply and demonstratives would have to be brought back in. Similar arguments can be devised for other demonstratives of places, as well as demonstratives of time, persons,⁴⁷ directions, things and spatial forms. Hence, in so far as knowledge of empirical things is concerned, all non-demonstrative identifications ultimately have to be grounded on demonstrative identifications.⁴⁸

The gist of these arguments is that, if we abstract from any spatio-temporal relations things may have to us and rely exclusively on non-demonstrative descriptions,⁴⁹ we remain incapable of saying whether a given hand is right or left, or what is meant by an utterance of ‘now’, and other things of this nature. In the analytic literature, the point is often made in linguistic terms. The claim is then that demonstrative *expressions* are not reducible to (or translatable without loss into) non-demonstrative expressions such as proper names and non-demonstrative descriptions. The claim is not that all demonstratives are essential (in the sense that none of them can be reduced to others),⁵⁰ but just that information conveyed by a demonstrative cannot be conveyed by a non-demonstrative expression. In any case, the linguistic claim about the irreducibility of demonstrative expressions rests on the epistemic claim about the insufficiency of conceptual capacities for the identification of particulars.⁵¹

This is only part of what Kant was up to, however. Also, and perhaps more importantly for Kant, the identification and recognition of forms and orientations cannot be carried out by conceptual capacities alone. And even if concepts were somehow to yield identification of particular places and things, that would still not give us spatial forms and orientations. Discussions of demonstrative identification provide us with a useful and clear model for approaching Kant’s remarks, and can be viewed as corroborating his claims. But Kant’s epistemological

remarks about the cognition of spatial forms and orientations are only hinted at in those discussions. Perhaps those cognitions may, too, be thought of in terms of demonstrative identification, in so far as they are cognitions of things given to us in perception. Then again, the things we perceive are spatially structured in certain ways, and we cannot but perceive them as such. This is an aspect of Kant's thought that is at best only hinted at in the literature on demonstratives. Whether we go on and think of those structures as given and having an 'inner ground', or think of them as grounded in the ways objects affect us, we will once again be brought back to the Kantian remarks with which we began.⁵²

Notes

For comments and criticisms of earlier and later versions I am grateful to Sven Bernecker, Andrew Blom, Mario Caimi, Fabian Domingues, Tiago Falkenbach, Renato Fonseca, José Guerzoni, Nick Huggett, David Potts, Valério Rohden, César Schirmer dos Santos, Márcio Teixeira, João Carlos Brum Torres and three anonymous referees from this journal. I am particularly indebted to Paulo Faria, my teacher and master's dissertation adviser at UFRGS (Brazil), with whom I seriously tackled the issues here discussed for the first time, and to my Kant teacher at UIC, Daniel Sutherland, for his illuminating lectures and a careful reading of the penultimate draft. I would also like to thank the CAPES Foundation (Brazil) for the support of research through a fellowship grant (BEX 1717/00-6).

- ¹ Peter Remnant, 'Incongruent counterparts and absolute space', *Mind*, 72 (1963), 393-9; reprinted in James Van Cleve and Robert Frederick (eds), *The Philosophy of Right and Left: Incongruent Counterparts and the Nature of Space* (Dordrecht: Kluwer, 1991), pp. 51-9. Kant's argument for absolutism (in *Von dem Ersten Grunde des Unterschiedes der Gegenden im Raume* (1768), 2: 377-83) is discussed in several of the papers in Van Cleve and Frederick. Kant's work is translated into English in *Theoretical Philosophy 1755-1770* in the Cambridge edition of the works of Immanuel Kant (Cambridge: Cambridge University Press, 1992). All citations of Kant's pre-Critical works are taken from this edition and references to the Academy edition are given by volume and page number. See also Felix Mühlhölzer, 'Das Phänomen der inkongruenten Gegenstücke aus Kantischer und heutiger Sicht', *Kant-Studien*, 83 (1992), 436-53; Paul Rusnock and Rolf George, 'A last shot at Kant and incongruent counterparts', *Kant-Studien*, 86 (1995), 257-77; Nick Huggett, 'Reflections on parity non-conservation', *Philosophy of Science*, 67

(2000), 219–41, and ‘Kant and the chiral arguments for substantialism’ (forthcoming as a chapter of a book by the author); Carl Hoefer, ‘Kant’s hands and Earman’s pions: chirality arguments for substantial space’, *International Studies in the Philosophy of Science*, 14 (2000), 237–56.

² According to some, such a defence would be out of place, since Kant never fully embraced Newtonian absolutism. See for example, David Walford, ‘Towards an interpretation of Kant’s 1768 *Gegenden im Raume* Essay’, *Kant-Studien*, 92 (2001), 407–39, esp. 433–6.

³ *De Mundi Sensibilis atque Intelligibilis Forma et Principiis* (henceforth: *1770 Dissertation*), 2: 403.

⁴ See section 2 below.

⁵ Hans Vaihinger suggests that Kant might have learned about them from a published work by Segner (1741) to which he had access. See Vaihinger, *Commentar zu Kants Kritik der reinen Vernunft* (Stuttgart, 1892; reprinted New York: Garland, 1976), vol. 2, p. 531 n. An anonymous reviewer observed that ‘Two of the Archimedean polyhedra – the so-called “snub cube” and the “snub dodecahedron” – have a “left-” and “right-handed” version’, and that ‘[i]n Part Four of the *Principles of Philosophy*, Descartes exploits the fact that screws can have oppositely winding threads in order to explain magnetic attraction and repulsion.’

⁶ See Leibniz, ‘De Analysis Situs’ (1679) in C. I. Gerhardt (ed.), *Die mathematischen Schriften von G. W. Leibniz*, vol. 5 (Halle: H. W. Schmidt, 1849), p. 179; and Wolff, *Mathematisches Lexicon* (Halle, 1751; reprinted Hildesheim: Olms, 1983), p. 1179. See also Rusnock and George, ‘Last Shot’, pp. 260–2, for further references and an excellent exposition and discussion of the topic. The next few paragraphs draw considerably from their paper.

⁷ Contrary to what Kant suggested, however, this is not a universal rule: there are objects that are not divisible into two symmetrical halves whose counterparts are congruent. Take, for example, a rectangular sheet of paper, pick two diagonally opposing corners, say, the top right and the bottom left, and fold them 90 degrees in opposite directions from the plane of the sheet of paper – that is, fold one of the corners up and the other down 90 degrees. The resulting object will not be divisible into two symmetrical halves (it will have no plane of mirror symmetry) but will nonetheless have a congruent counterpart. For an illustration, see Martin Gardner, *The New Ambidextrous Universe* (3rd rev. edn, New York: W. H. Freeman & Co, 1991), p. 14.

⁸ Later, in the nineteenth century, it was pointed out (originally by Ferdinand Möbius, in ‘On higher space’ (1827), reprinted in Van Cleve and Frederick, *Philosophy*, pp. 39–41) that things that are incongruent in a three-dimensional space can be turned around in a four-dimensional space so as to be enclosed in the same limits as their counterparts. In general, any n -dimensional object can be flipped over in an $n+1$ -dimensional

space so as to be enclosed in the same limits as its counterpart. Also movement along a non-orientable space (for example, a Möbius strip, or a Klein bottle) may change the handedness of a particular object. But these mathematical possibilities were physically inconceivable to eighteenth-century thinkers like Kant, and even to nineteenth-century thinkers such as Möbius himself. ‘Space’ then did not mean a mere mathematical construct, but at once the space of physics, geometry and of ordinary experience, and it was conceived as Euclidean and three-dimensional (see Roberto Torretti, *Philosophy of Space from Riemann to Poincaré* (Boston: D. Reidel, 1978), introduction). Only later was a distinction between physical, phenomenological and mathematical spaces introduced. This explains why Kant only mentions as examples of incongruent counterparts things that cannot be moved and turned around in Euclidean three-dimensional space so as to occupy the same limits: ‘If two figures drawn on a plane surface are equal and similar, then they will coincide with each other. But the situation is often entirely different when one is dealing with corporeal extension, or even with lines and surfaces, not lying on a plane surface’ (2: 381). Kant never mentions flat two-dimensional objects or one-dimensional objects as having incongruent counterparts precisely because they can be turned over in three-dimensional space so as to coincide with their counterparts. For him, three-dimensionality was an actual feature of *the* (unique) space. One- and two-dimensional spaces were but limitations of *the* (three-dimensional) space, and higher dimensional spaces were just physically inconceivable (in his 1747 essay on Living Forces, for example, Kant writes that these higher dimensional spaces, if possible, would ‘not belong to our world’). To regard Kant’s notion of an incongruent counterpart as limited to oriented spaces of a particular dimensionality – instead of referring to *the* (unique) space – is an anachronism somewhat diffused in the literature (for example, in Van Cleve and Frederick, *Philosophy*, pp. 8, 22–3; and Graham Nerlich, ‘Hands, Knees, and Absolute Space’, *ibid.*, esp. pp. 158–9; see also pp. lxix–lxx of the introduction to the English translation of Kant’s 1768 essay in *Theoretical Philosophy 1755–1770*).

⁹ ‘Last shot’, pp. 260–1.

¹⁰ ‘*Similar segments of circles* are those which admit equal angles, or in which the angles are equal to one another’ (Euclid, *Elements*, bk III, def. 11, in T. L. Heath (ed.), *The Thirteen Books of Euclid’s Elements* (New York: Dover, 1956)); ‘*Similar rectilineal figures* are such as have their angles severally equal and the sides about the equal angles proportional’ (bk VI, def. 1); ‘*Similar plane and solid numbers* are those which have their sides proportional’ (bk VII, def. 21); ‘*Similar cones and cylinders* are those in which the axes and the diameters of the bases are proportional’ (bk XI, def. 24).

¹¹ ‘In undertaking an explanation of quality or form, I have learned that the

matter reduces to this: things are *similar* which cannot be distinguished when observed in isolation from each other. Quantity can be grasped only when the things are actually present together or when some intervening thing can be applied to both. But quality presents something to the mind which can be known in a thing separately and can then be applied to the comparison of two things without actually bringing the two together either immediately or through the mediation of a third object as a measure' in C. I. Gerhardt (ed.), *Die philosophischen Schriften von G. W. Leibniz* (Berlin: Hale, 1875), vol. 5, p. 179; English translation in L. Loemker (ed.), *G. W. Leibniz: Philosophical Papers and Letters* (Chicago: University of Chicago Press, 1956), vol. 1, p. 392.

¹² '... *similar* are those things that cannot be discerned in isolation' (. . . ut *similia* sint, quæ singulatim observata discerni non possunt) (Leibniz, 'De analysis Situs' (1679), in *Die mathematischen Schriften*, vol. 5, p. 180).

¹³ 2: 382. The surface that limits a left hand could of course be pulled inside out so as to 'serve as a boundary to limit' its right counterpart. This is a rather elementary objection to Kant's formulation, and I should like to thank an anonymous reviewer for pointing it out. I guess Kant's reply would be that we ought *not* to take 'surface' here as meaning something like a very thin layer wrapped around a body. Rather, it is meant to refer to the two-dimensional space which limits a three-dimensional body. And the point is merely that that space cannot be moved rigidly (that is, without stretching or bending) so as to overlap the two-dimensional limits of its incongruent counterpart.

¹⁴ Rusnock and George, 'Last shot', p. 266, maintain that Kant's usage of 'inner ground' in the 1768 essay is indeed equivocal. However, the equivocation only shows up if one reads Kant's distinction as mapping onto Leibniz's, in the sense that all qualitative Leibnizian characteristics would count as having an inner ground for Kant, and all quantitative Leibnizian characteristics except the orientation of objects such as hands would count as having an outer ground for Kant. On this reading, Kant would indeed say that orientation has an inner ground sometimes in the sense that it is not discernible in isolation, and sometimes in the sense that it is neither magnitude nor location. There is, however, no need to read this equivocation into the text.

¹⁵ 2: 380. For an excellent discussion of Kant's use of these examples (as well as of Kant's 1768 essay in general), see Roberto Torretti, *Manuel Kant* (Santiago de Chile: Ediciones de la Universidad de Chile, 1967), pp. 120ff.

¹⁶ 'On the other hand, where a given rotation can be attributed to the course of those two celestial bodies [sun and moon] – *Mariotte* claims to have observed such a law operating in the case of the winds: he maintains that from new to full moon the winds tend to change their direction clockwise through all the points of the compass – then this circular movement must rotate in the opposite direction in the other hemisphere. And this is

something which *Don Ulloa* claims to have found actually confirmed by his observations in the south seas' (2: 380).

¹⁷ 'There is only one way in which we can perceive that which, in the form of a body, exclusively involves reference to pure space, and that is by holding one body against other bodies' (2: 383).

¹⁸ 2: 383. There is some controversy in the literature about this reference to a human body. Remnant, 'Incongruent counterparts', 398, finds an inconsistency in Kant's argument: 'We can now see where Kant's own argument goes wrong: it involves the inconsistency of maintaining that it is impossible to say of a hand, considered entirely in isolation from everything else, whether it is right or left, while assuming that it would be possible to say of a handless body, considered by itself, which was its right side and which its left'. See also Nick Huggett, *Space from Zeno to Einstein* (Cambridge, MA: MIT Press, 1999), pp. 208–12, and Rusnock and George, 'Last shot', pp. 265–6, and Oliver Pooley, 'Handedness, parity violation and the reality of space', in K. Brading and E. Castellani (eds), *Symmetries in Physics* (Cambridge: Cambridge University Press, 2003), pp. 258–63. But perhaps the problem with Kant's argument lies – contrary to what these authors suggest – not in the thought-experiment itself, but in the inference Kant draws from it, to the effect that Leibniz's conception of space must be wrong. Although it is true that it hardly makes any sense to speak of a hand being right or left in the absence of any other object to which it could be compared, Kant's thought-experiment does not presuppose that it does. Rather, the experiment relies merely on the fact that a hand has to have a form if it is to count as a hand at all. To be sure, a human body, too, must have a form, and even if we cannot say which of its sides is the right side and which is the left side, it is still the case that a lone hand will only fit one of those sides, not both.

¹⁹ For a defence of relationism, and further references, see Nick Huggett, 'Geometry and topology for relationists' (this is a chapter in a forthcoming book by the author), also, Huggett, 'Mirror symmetry: what is it for relational space to be orientable?' (<http://philsci-archive.pitt.edu/archive/00000767/>).

²⁰ For a recent summary of these accounts, and further references, see Pooley, 'Handedness', p. 253.

²¹ (2: 403). Here is the analogous argument for the one-dimensional case of time: 'If you think of two years, you can only represent them to yourself as joined to one another by some intermediate time. But among different times, the time which is *earlier* and the time which is *later* cannot be defined in any way by any characteristic marks which can be conceived by the understanding, unless you are willing to involve yourself in a vicious circle' (2: 399). Note that in both cases it is orientation that is at stake: the orientation of a three-dimensional spatial form in 2: 403, and of the flow of time in 2: 399.

- 22 'hic non nisi quadam intuitione pura diversitatem, nempe discongruentiam, notari posse' (1770) (2: 403).
- 23 For example, by C. F. Gauss, 'Theoria residuorum biquadraticorum. Commentatio secunda', (1831), in *Werke* (Göttingen, 1863. reprinted: Hildesheim: Olms, 1975), vol. 2, p. 177; F. Klein, *Elementary Mathematics from an Advanced Standpoint* (New York: Dover, 1948), pp. 39–43; H. Weyl, *Philosophy of Mathematics and Natural Science* (Princeton: Princeton University Press, 1963), pp. 78–84; Jonathan Bennett, 'The difference between right and left', *American Philosophical Quarterly*, 7 (1970) – reprinted in Van Cleve and Frederick, *Philosophy*, pp. 97–130; Mühlhölzer, 'Das Phänomen'; Rusnock and George, 'Last shot'; and a few others in Van Cleve and Frederick, *Philosophy*.
- 24 'Das Phänomen', p. 452.
- 25 Similar indications, by other authors, on how to distinguish incongruent counterparts without intuitions can be found in the other papers mentioned in note 23 above.
- 26 'Last shot', p. 273.
- 27 'It is the business of philosophy to analyse concepts which are given in a confused fashion, and to render them complete and determinate' (*Untersuchung über die Deutlichkeit der Grundsätze der natürlichen Theologie und der Moral* (1764): 2: 278).
- 28 *Gegend* is commonly translated as *region*. In this case, as argued by David Walford and Rusnock and George, *direction* seems to be a more adequate rendering of the term. See David Walford's notes to his translation of the 1768 essay (in *Theoretical Philosophy 1755–1770*, pp. 456–7), as well as his 'Towards an interpretation of Kant's 1768 *Gegenden im Raume* Essay', *Kant-Studien*, 92 (2001), 407–39; also Rusnock and George, 'Last shot', pp. 269–72.
- 29 *Metaphysische Anfangsgründe der Naturwissenschaft* (1786): 4: 484.
- 30 Rusnock and George, 'Last shot', p. 274.
- 31 See *KrV*, B VIII.
- 32 Rusnock and George, 'Last shot', p. 274.
- 33 2: 273–301. The methods of philosophy and mathematics are contrasted throughout the essay. See, for example, the following passage: 'geometers acquire their concepts by means of *synthesis*, whereas philosophers can only acquire their concepts by means of *analysis* – and that completely changes the method of thought' (2: 289). Similar remarks can be found in *KrV* A713/B741ff.
- 34 *Ibid.* The same point is made in *Was heisst: Sich im Denken orientieren* (1786): 8: 134–5.
- 35 See Kant's *Logik* (Jäsche), §1, where concepts are characterized as *representatio per notas communes*.
- 36 Similarly for 'after' and other temporal notions: 'I only understand the meaning of the little word *after* by means of the antecedent concept of time' (2: 399).

- ³⁷ Later, in the *Critique of Pure Reason*, Kant would write that ‘reason demands . . . that no species be regarded as in itself the lowest; for since each species is always a concept that contains in itself only what is common to different things, this concept cannot be thoroughly determined, hence it cannot be related to an individual’ (A655/B683. English translation by Guyer and Wood (Cambridge: Cambridge University Press, 1998)).
- ³⁸ A similar point seems to underlie Lorne Falkenstein’s discussion of the perception of a triangle drawn on a sheet of paper in *Kant’s Intuitionism* (Toronto: University of Toronto Press, 1995): if each point on the sheet is designated by an ordered triple, where the first two elements of the triple are Cartesian coordinates and the third indicates the colour of that particular point, then although ‘each point on the sheet is the subject-matter for a distinct thought, there is nowhere a thought of the appearance of the sheet as single whole, and there is therefore a sense in which the sheet has not been perceived’ (p. 246). Incongruent counterparts are a little more complex in this respect since even if we were somehow capable of grasping part-whole relations from a reading of the ordered triples, that would still tell us nothing about orientation. Falkenstein points out that the form of a triangle cannot be given by taxonomy, mereology is also required. Incongruent counterparts show that in some cases not even that suffices: part-whole relations tell us nothing about orientation. Falkenstein goes on to say that ‘[w]hen I draw a triangle on paper, there is a sense in which the “space” for my activity already exists. . . . The order of the points is originally fixed by the paper, not by my own decision about how to draw . . . All I can do . . . is choose how to run through an order of points that is already given. I cannot create or define the order itself or, to put the point more technically, I cannot change its topology . . . as I please’ (p. 247). Yes, but also the geometry (not just the topology) cannot be changed as one pleases.
- ³⁹ The problem is similar to the following: given a description of the colours blue and red in terms of light-rays and their respective wave-lengths, we can perfectly well distinguish one from the other. But nothing in those descriptions allows us to know what those colours look like. The latter knowledge requires a non-conceptual capacity, such as the one paradigmatically exemplified in perceiving red and blue things. This is also corroborated by the fact that it is possible to describe light-rays whose appearance we cannot even imagine (for example, ultra-violet rays). The analogy here is only partial, however, since a conceptual description plus a sensation (the ‘matter’ of an intuition) could perhaps suffice for the identification and recognition of a colour, but does not suffice for the identification and recognition of a spatial form. Sensations only give us indications of the sensory properties of an object (colour, taste, odour, etc.), but not of ‘extensive magnitudes’ (see *KrV*, A167/B209). And it is doubtful, to say the least, that spatial properties (extension, shape,

continuity, orientation, etc.) could somehow be inferred from sensory properties. In 1768 (2: 380) and again in 1786 (8: 131ff.) Kant does speak of a 'feeling (*Gefühl*) of right and left', but there he means not a feeling of spatial forms *per se*, but merely a feeling of a difference in the sensations we have on each side of our bodies: the right hand is stronger in most people, the left eyes and ears are said to be more sensitive, etc.

40 A good explanation for the fact that they appear in the latter but not in the former can be found in the different methods of exposition, analytic and synthetic, adopted in the *Prolegomena* and in the first *Critique*, respectively. The plainly accessible fact that there are incongruent counterparts is suited for an analytical mode of presentation, which begins with 'something already known to be dependable, from which we can go forward with confidence and ascend to the sources, which are not yet known, and whose discovery not only will explain what is known already, but will also exhibit an area with many cognitions that will arise from these same sources' (4: 274–5). English translation by Gary Hatfield, *Prolegomena to any Future Metaphysics* (Cambridge: Cambridge University Press, 1997)). The synthetic mode adopted in the *Critique*, on the other hand, proceeds 'by inquiring within pure reason itself, and seeking to determine within this source both the elements and the laws of its pure use, according to principles' (4: 274). Hence, something like an argument from incongruent counterparts, which one might otherwise expect to find in the sections on space of the Transcendental Aesthetic, is nowhere to be found. None of the arguments of the Transcendental Aesthetic begin with empirical facts that are 'known to be dependable'; they begin instead with general notions and principles. On Kant's methods of presentation, see Mario Caimi, 'About the argumentative structure of the Transcendental Aesthetic', *Studi Kantiani*, 9 (1996), 27–46, and Lisa Shabel, 'Kant's "Argument from Geometry"' (*Journal of the History of Philosophy*, 42, 2004, 195–215).

41 4: 484. Incongruent counterparts are also briefly discussed in the first few paragraphs of the 1786 essay *Was heisst: Sich im Denken orientieren* (8: 131ff.).

42 See, for example, Arthur Prior, 'Thank goodness that's over!', *Philosophy*, 34 (1959), 12–17, Peter Strawson, *Individuals* (London: Methuen, 1959), part 1, and Richard Gale, 'Tensed statements', *Philosophical Quarterly*, 12 (1962), 53–9. (The examples to be used were elaborated upon the ones discussed by these authors.) In 'Knowledge by acquaintance and knowledge by description', Bertrand Russell seems to rely on a similar reasoning (see Paulo Faria, 'Discriminação e Afecção', in E. Marques (ed.), *Verdade, Conhecimento e Ação* (São Paulo: Loyola, 1999), pp. 145–59). More recent discussions of demonstratives can be found in Stephen Schiffer, 'The basis of reference', *Erkenntnis*, 13 (1978), 171–206; David Lewis, 'Attitudes *de dicto* and *de se*', *Philosophical Review*, 88 (1979), 513–43; John Perry,

‘The problem of the essential indexical’ (1979), in Nathan Salmon and Scott Soames (eds), *Propositions and Attitudes* (Oxford: Oxford University Press, 1988); Robert Stalnaker, ‘Indexical belief’, *Synthese*, 49 (1981), 129–51; Roderick Chisholm, *First Person: An Essay on Reference and Intentionality* (Minneapolis: University of Minnesota Press, 1982); David Austin, *What’s the Meaning of “This”?* (Ithaca, NY: Cornell University Press, 1990); David Martens, ‘Demonstratives, descriptions, and knowledge: a critical study of three recent books’, *Philosophy and Phenomenological Research*, 54 (1994), 947–63; and, by the same author, ‘Indexicals’, *Stanford Encyclopedia of Philosophy* (2001), available online at <http://plato.stanford.edu/entries/indexicals/>.

43 ‘Demonstrative identification’ here means forms of identification of particular things in which the thing identified varies from context to context even when the words and gestures employed in the demonstration remain the same, that is, are used with the same meanings and intentions. Typically, an ostensive identification, where someone says something like ‘this’ or ‘that’ and points to a certain thing, counts as a demonstrative identification. Also counted are expressions like ‘here’, ‘now’, ‘I’, ‘you’, and ‘to my right/left’, ‘up’, etc., accompanied by gestures or not.

44 The description need not be correct for the identification to obtain. On this and related topics, see Keith Donnellan, ‘Reference and definite descriptions’, *Philosophical Review*, 75 (1966), 281–304.

45 The following passage by William James illustrates the point: ‘If we take a cube and label one side *top*, another *bottom*, a third *front*, and a fourth *back*, there remains no form of words by which we can describe to another person which of the remaining sides is *right* and which is *left*. We can only point and say *here* is right and *there* is left, just as we should say *this* is red and *that* blue’ (*The Principles of Psychology*, ch. 20, ‘The Perception of Space’ (New York: Dover, 1890, 1918), vol. 2, p. 150).

46 See Gale, ‘Tensed statements’, pp. 56–7.

47 Here is an interesting example on the use of ‘I’: ‘An amnesiac, Rudolf Lingens, is lost in the Stanford library. He reads a number of things in the library, including a biography of himself, and a detailed account of the library in which he is lost . . . He still won’t know who he is, and where he is, no matter how much knowledge he piles up, until that moment in which he is ready to say, “*This* place is aisle five, floor six, of Main Library, Stanford. *I* am Rudolf Lingens.” ’ (John Perry, ‘Frege on demonstratives’, *Philosophical Review*, 86 (1977), 492).

48 Nelson Goodman objected to a similar argument that a translation without loss into a language without time indexicals is indeed possible as long as a calendar-watch (or, in the case here discussed, where only the identification of places is relevant, something like a GPS device), is available. He argues in *The Structure of Appearance* (3rd edn, Dordrecht: D. Reidel, 1977), p. 269, that consulting such a device is as legitimate, as far as translations

go, as consulting a dictionary when one is translating into or from a foreign language. But how could one know (without demonstrative identifications) that the device works? How can one know whether a watch shows the current time, or whether a GPS device indicates the place where one is, without ultimately relying on some information, gathered from an independent source, of the type: 'It is now 5 p.m. and that is also what my watch shows' or 'We are now at 50 degrees west of Greenwich, 30 degrees south of the Equator, and that is also what my GPS shows'? In fact, Goodman's argument can run the other way around also. Information conveyed by dictionaries, too, ultimately depend on a lexicographer's demonstrative identification of foreign utterances. Using watches and GPS devices to eliminate spatio-temporal demonstratives from our utterances does not entail that the knowledge expressed by such utterances is independent from other pieces of information that must be expressed with the aid of demonstratives. The link between what a watch or a GPS device displays and the time–location one is at cannot be established by a description that does not contain demonstratives. Ordinarily we just take that link for granted. But it obviously must be established at some point. Likewise, the use of dictionaries for translations relies on demonstrative identification of foreign utterances and utterance patterns: the link between the written signs it contains and the actual noises and gestures foreigners make while speaking cannot be established solely by means of descriptions that do not contain demonstratives.

49 That is, descriptions that do not themselves contain demonstratives.

50 To be sure, some demonstratives are essential in that sense. Perry, 'Problem', p. 96, suggests that it is plausible to think that 'I' and 'now' are in fact the only essential demonstratives. 'Tomorrow' (the day after the one *I* am in *now*), 'you' (the person *I* am addressing *now*) and all other demonstratives would be expressible in terms of 'I' and 'now'. However, if we are to take up Perry's suggestion, then at least three other demonstratives, all of them demonstratives of orientation – one for each spatial dimension; 'right', 'up', and 'front', for example – would have to be included in the list. As the passage by James quoted in note 45 above indicates, there is no way of rendering the contents of 'right', 'up' and 'front' in terms of 'I' and 'now'.

51 Hence, also insufficient for determining the extension of a term. See, for example, Hilary Putnam's 'The meaning of "Meaning"', in *Philosophical Papers*, vol. 2 (Cambridge: Cambridge University Press, 1975), pp. 215–71.

52 The relation between Kant's remarks and contemporary analytic themes is discussed by Palle Yourgrau, 'The path back to Frege', *Proceedings of the Aristotelian Society*, 87 (1987), 169–210, esp. pp. 101–3, and João Carlos Brum Torres, 'Cognição intuitiva e pensamento *de re*', *Analytica*, 4 (1999), 33–63.