A PUZZLE ABOUT INCONGRUENT COUNTERPARTS AND THE *CRITIQUE OF PURE REASON*

BY

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Abstract: Kant uses incongruent counterparts in his work before and after 1781, but not in the first *Critique*. Given the relevance that incongruent counterparts had for his thought on space, and their persistence in his work during the 1780s, it is plausible to think that he had a reason for leaving them out of both editions of the *Critique*. Two implausible conjectures for their absence are here considered and rejected. A more plausible alternative is put forth, which explains that textual absence as a result of the synthetic method of presentation intended for the *Critique*.

Kant defines an incongruent counterpart as a "body which is exactly similar and equal to another, but cannot be enclosed in the same limits as that other."[1] He was not the first to notice the existence of such objects, but was indeed the first to extract significant philosophical lessons from them.[2] Incongruent counterparts appear several times and in important contexts throughout Kant's writings from the 1760s through the 1780s.[3] Recently commentators have pointed out that they played a crucial role in the development of Kant's thought on space.[4]

The question I wish to discuss here is a lesser one: given that incongruent counterparts were used by Kant in his writings before and after 1781, and assuming that they were in fact instrumental in the development of his thought as commentators have recently claimed, then why are they nowhere to be found in Kant's most important work, the *Critique of Pure Reason*? The question has long puzzled commentators, who have tentatively put forth a few conjectures. Not one of them is cogent. In what follows I

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briefly review Kant's uses of incongruent counterparts before and after 1781 (section 1), consider and reject two explanations for the absence of incongruent counterparts from the first *Critique* (section 2), and conclude with a conjecture of my own (sections 3 and 4). I shall leave aside here the question of how best to interpret the various arguments from incongruent counterparts, and whether they are sound.[5]

1. Kant's uses of incongruent counterparts

Within Kant's complete works, the notion of "congruence" makes its first appearance in 1763, in some lecture notes taken by Herder. In those notes, Herder jotted down a version of the thesis held by Leibniz and Wolff according to which all similar and equal things are congruent, and immediately afterwards restricts the thesis to things that lie in a plane:

> *Congruentia*. Those [things] that are equal and similar to each other . . .
> The concepts of congruence are extended in mathematics. Equal and similar [things] are not congruent unless they lie in a *plane*.[6]

In his prize essay for the Berlin Academy, published in 1764, Kant identifies the source of Lebniz's and Wolff's mistake in a misapplication of "the analytic method" for defining a concept to the mathematical concept of similarity.[7] Wolff, he wrote, "looked at it [the concept of similarity] with a philosophical eye" (2: 277) and took similarity as defined for certain pairs of objects to be sufficient for a definition of the concept of similarity in general. But mathematics and philosophy, Kant notes, have very distinct methods:

> In mathematics I begin with the definition of my object, for example, of a triangle, or a circle, or whatever. In metaphysics I may never begin with a definition. Far from being the first thing I know about the object, the definition is nearly always the last thing I come to know. In mathematics, namely, I have no concept of my object at all until it is furnished by the definition. In metaphysics I have a concept which is already given to me although it is a confused one. (2: 283)

According to Kant, Wolff treated the mathematical definition of similarity (as it applies to specific pairs of objects) as if it were a philosophical definition. Wolff would thus have extended the mathematical notion of similarity to all geometrical objects "with a view to subsuming the geometrical concept of similarity under the general concept" (2: 277). Kant also notes that by using the mathematical method in philosophy, Leibniz made the converse mistake: He "imagined a simple substance which had nothing but obscure representations, and he called it a *slumbering monad*. But in doing so, he did not define the monad. He merely invented

it, for the concept of a monad was not given to him but created by him" (ibid.). I shall not discuss in any detail here what Kant meant by those differences in method, but merely state that the passages just quoted refer to methods for defining a concept, and that awareness of incongruent counterparts was likely to have clued Kant on Wolff's mistake.[8] Assuming this is true, then incongruent counterparts were important for the development of Kant's philosophical method already in the early 1760s.

Kant further distanced himself from Leibniz and Wolff with his 1768 essay "Concerning the ultimate ground of the differentiation of directions in space," and again incongruent counterparts played an important role – this time, more explicitly so. In that essay Kant introduces a general method for constructing counterparts by reflection in a plane (2: 382), and demonstrates that incongruent counterparts are counterexamples to the Leibnizian thesis mentioned above (that all equal and similar things are congruent). He infers from that that the Leibnizian conception of space must be wrong, for it would entail the absence of a difference (between pairs of incongruent counterparts) where there clearly is one: right hands do not fit into left gloves, and vice-versa. Against Leibniz, Kant offers a proof of the reality of absolute space. The details of the argument do not interest us here;[9] it suffices to notice that by putting forth a proof of the reality of absolute space Kant was rejecting the Leibnizian conceptions of space which he had entertained up until then.[10] His views on the nature of space would change again soon afterwards, as is well known, but this first explicit divergence with Leibniz seems to have been crucial for the latter development of his own original ideas. Yet, although his views on the nature of space would later change, his analysis of incongruence itself would remain the same from 1768 onwards.

The *1770 Dissertation* already contains much of what would later make up the "Transcendental Aesthetic."[11] In particular, it already states that space is an a priori intuition and that it is "the fundamental form of all outer sensation" (2: 402). To argue for the latter claim, Kant maintains that space is a pure intuition, which "can easily be seen in the axioms of geometry, and in any mental construction of postulates, even of problems" (ibid.). He gives a few examples of some characteristics of space that are affirmed by the axioms and postulates of (Euclidian) geometry and which cannot be "derived from any universal concept of space." Incongruent counterparts provide one of those examples:

> Which things in given space lie in one direction and which things incline in the opposite direction cannot be described discursively nor reduced to characteristic marks of the understanding by any astuteness of the mind. Thus, between solid bodies which are perfectly similar and equal but incongruent, such as the left and right hands (in so far as they are conceived only according to their extension), or spherical triangles from opposite hemispheres, there is a difference, in virtue of which it is impossible that the limits of their extension should coincide – and that, in spite of the fact that, in respect of everything which

may be expressed by means of characteristic marks intelligible to the mind through speech, they could be substituted for one another. It is, therefore, clear that in these cases the difference, namely the incongruity, can only be apprehended by a certain pure intuition. (2: 403)

Incongruent counterparts thus figure in an argument to the effect that space is a pure intuition. This is quite a shift from the 1768 essay, where incongruent counterparts were called upon to prove the reality of absolute space. However, there are continuities in Kant's uses of incongruent counterparts: In both texts, they appear as examples of things that *cannot* be fully apprehended by conceptual means alone, but only relative to something unique ("absolute space" in the 1768 essay; "the form of outer sensations" in 1770). In both arguments Kant attempts to explain how similar and equal things can nonetheless be incongruent. In both cases the argument begins with the statement of plainly accessible cognitions (the shapes of human hands, for example), and seeks to explain their possibility.

The sections on space and time in the *1770 Dissertation* were probably used in early drafts of the *Critique of Pure Reason*. The sections on Space (§15) of the *Dissertation* more or less parallel the ones in the "First Section" of the "Transcendental Aesthetic." Hence, one could reasonably expect to find the argument from incongruent counterparts somewhere in the "Conclusions from the Above Concepts" section of the "Transcendental Aesthetic." But, as we know, they are nowhere to be found there or elsewhere in the *Critique*.

It is likely that Kant had a reason for leaving them out, given the importance they had for the development of his conception of space. He certainly did not forget about them. Just a couple of years after the publication of the first edition of the *Critique*, he uses incongruent counterparts again, in the *Prolegomena*:

If two things are fully the same (in all determinations belonging to magnitude and quality) in all the parts of each that can always be cognized by itself alone, it should indeed then follow that one, in all cases and respects, can be put in the place of the other, without this exchange causing the least recognizable difference. In fact this is how things stand with plane figures in geometry; yet various spherical figures, notwithstanding this sort of complete inner agreement, nonetheless reveal such a difference in outer relation that one cannot in any case be put in the place of the other; e.g. two spherical triangles. . . . (4: 285)[12]

Kant also refers to incongruent counterparts in two other works of the period, the *Metaphysical Foundations of Natural Science* (4: 483–484) and in the essay "What is Orientation in Thinking?" (8: 134 f.), both published in 1786. Although the arguments from incongruent counterparts in these texts from the 1780s differ from the one in the *1770 Dissertation*, the mere fact that they were there used shows that incongruent counterparts were still very much present to Kant's mind well into the critical period. Yet, he also left them out of the revised second edition of the *Critique* (1787).

2. *Conjectures*

Norman Kemp Smith has an explanation.[13] He claims that Kant rejected the argument from incongruent counterparts because it contained a "dogmatic premise" (something Kant would only have noticed after the *Prolegomena* and the *Metaphysical Foundations of Natural Science* were published). This would only explain the absence of incongruent counterparts from the second edition of the *Critique*, but not their absence from the first edition, as Kemp Smith himself acknowledges. Still, he contends that Kant found out that the arguments in the *Prolegomena* and in the *Metaphysical Foundations* were "entirely out of harmony with the teaching of the *Critique*" (p. 165). As Kemp Smith construes those arguments, they begin by stating that certain bodies, "conceptually considered, can be absolutely identical, and yet for intuition remain diverse" (p. 164). This would show that "such bodies and the space within which they fall are not independent existences" (ibid.). Hence, they cannot be things in themselves; if they were, pure understanding would be capable of cognizing them. Therefore, they must be subjective and ideal.

The dogmatic premise of the argument as construed by Kemp Smith lies in the assumption that pure understanding can in fact cognize things in themselves. This was a doctrine that Kant inherited from the Leibnizian tradition, and which he still maintains in the *1770 Dissertation*, but later rejects. According to Kemp Smith, those pre-critical thoughts would have lingered on "unconsciously" in the argument from incongruent counterparts that Kant used in the 1780s. In the *Prolegomena* Kant does indeed say that incongruent counterparts "are surely not representations of things as they are in themselves, and as the pure understanding would cognize them" (4: 286). Likewise, in the *Metaphysical Foundations* he says that "space in general does not belong to the properties or relations of *things in themselves*, which would necessarily have to be reducible to objective concepts, but rather belongs merely to the subjective form of our sensible intuition" (4: 484). According to Kemp Smith, Kant would have become aware of the inconsistency of these passages with the main doctrine of the *Critique* in the years between the writing of the *Metaphysical Foundations* and the writing of the second edition of the *Critique*, and accordingly suppressed it from all later writings.

However, even if Kemp Smith is right about the dogmatic premise, that would still not explain why Kant left incongruent counterparts out of the *first* edition of the *Critique*. Kemp Smith also fails to explain why the incongruent counterparts argument was not rewritten so as to leave out the dogmatic premise. In fact, Kant could have used the same argument that appears in the *1770 Dissertation*, which does not itself contain that premise, even though it is part of a book that does. The general goal of the argument that appears in the *Dissertation* is not to prove the ideality

of space, but rather that space is a form of all outer intuition. That particular argument does not require the "dogmatic premise," and could very nicely fit in with the statements made in the "Conclusions from the Above Concepts" section of the "Transcendental Aesthetic." Yet, Kant left the argument out.

Another author who has written on the topic is Jill Buroker (1981). She assigns great importance to incongruent counterparts in the development of Kant's thought and, accordingly, finds their absence from the *Critique* "extremely odd" (p. 85). She gives no positive explanation for the absence (at least not for their absence from the "Transcendental Aesthetic," which is presumably where one would expect to find it), and wonders whether Kant believed he just did not need the argument, given the strength of the ones that did make it into the book (p. 85 f.). She also wonders whether "his desire to present parallel treatments of space and time prevented him from arguing his theory from the nature of space without an analogous argument from the nature of time" (p. 86). This conjecture, however, is false. Kant did have an analogous argument for time, and he included it the *1770 Dissertation*:

> But among different times, the time which is *earlier* and the time which is *later* cannot be defined in any way by any characteristic marks which can be conceived by the understanding, unless you are willing to involve yourself in a vicious circle. The mind only discerns the distinction between them by a singular intuition. (2: 399)

Buroker, however, does think that a version of the incongruent counterparts argument found its way into the *Critique*; not into the "Transcendental Aesthetic," but into the "Amphibology of the Concepts of Reflection." That section, she says, contains a "compressed version of the incongruent counterparts argument" (p. 87):

> Through mere concepts, of course, I cannot think of something external without anything inner, for the very reason that relational concepts absolutely presuppose given things and are not possible without these. But something is contained in the intuition that does not lie at all in the mere concept of a thing in general; and this yields the substratum that cannot be cognized through mere concepts, namely a space that, along with everything that it contains, consists of purely formal or also real relations. . . . (A 284/ B 340)

Although this passage and the incongruent counterparts arguments are certainly connected, to say that they are versions of the same argument may be too much of a stretch. The various incongruent counterparts arguments have different premises and different conclusions, but they all begin with plainly available given cognitions, which is what the examples of incongruent counterparts are supposed to bring out. The passage from the "Amphibology" cited above, on the contrary, reasons from abstract principles (e.g. that "relational concepts absolutely presuppose given things

and are not possible without these") and infers from them the possibility of cognitions.

3. *The mode of presentation of the Critique*

In the opening pages of the *Prolegomena* (4: 263 and 4: 274 f.), Kant distinguishes what he calls the *analytic* and the *synthetic* modes of addressing a problem. He claims to have employed the latter in the *Critique* and the former in the *Prolegomena*. The question whether metaphysics is at all possible was answered synthetically in the *Critique*, "by inquiring within pure reason itself, and seeking to determine within this source both the elements and the laws of its pure use, according to principles" (4: 274). That answer "tried to develop cognition out of its original seeds without relying on any fact whatever" (ibid.). Hence, the answer began with general notions and principles, which were argued for without assuming any particular cognition. The *analytic*, or "regressive," mode of presentation used in the *Prolegomena*, on the other hand, relies:

> . . . on something already known to be dependable, from which we can go forward with confidence and ascend to the sources, which are not yet known, and whose discovery not only will explain what is known already, but will also exhibit an area with many cognitions that will arise from these same sources. (4: 274 f.)

Accordingly, the *Prolegomena* begins not with abstract principles, but with some (synthetic a priori) cognition that is taken for granted. Indeed, mathematics as a science is the starting point of the *Prolegomena*: "We have therefore some at least *uncontested* synthetic cognition a priori, and we do not need to ask whether it is possible (for it is actual), but only: *how it is possible*" (4: 275). The argument of the *Prolegomena* is therefore regressive. It seeks to unfold the conditions of possibility for such cognitions in general. It begins with cognitions which are taken for granted and works its way back to the principles on which such truths are necessarily grounded.

In the *Critique*, on the other hand, no cognitions are assumed or taken for granted at the outset. The starting point of the *Critique* is a set of general concepts and notions which are given to us, for example, the notions of "representation," "sensibility," and "understanding." These notions are introduced in the very first paragraph of the "Transcendental Doctrine of Elements":

> The capacity (receptivity) to acquire representations through the way in which we are affected by objects is called sensibility. Objects are therefore given to us by means of sensibility, and it alone affords us intuitions; but they are **thought** through the understanding, and from it arise concepts. (A 19/ B 33)

Hence, the *Critique* does not argue from given cognitions to general principles as the *Prolegomena* does, but from principles and general notions to the possibility of cognitions in general.[14] Kant maintains that the analytic mode of presentation is better suited when the aim is to persuade or educate the reader;[15] and for that reason it was used in the *Prolegomena*. But if the goal is to present systematically the results of an investigation, then the synthetic mode is better suited; and for that reason it was adopted in the *Critique*.

The distinction between the analytic and the synthetic modes of presentation is related to, but not the same as the distinction Kant makes in the 1764 prize essay between the analytic and the synthetic methods of defining a concept. The analytic and synthetic modes of presentation discussed in the *Prolegomena* are not methods of investigation. Instead, they are modes of presenting the results of an investigation that has already been done. Nevertheless, both the analytic *mode of presentation* and the analytic *method of investigation* have in common the fact that they begin with some particular cognition that is given. Thus, just like the analytic mode of presentation, the analytic method of investigation "begins with the conditioned and proceeds to principles (*a principiatis ad principia*)," whereas the synthetic method "goes from principles to consequences or from the simple to the composite. The former could also be called *regressive*, as the latter could *progressive*" (*Jäsche Logic*, §117).

Although the analytic/synthetic *modes of presentation* seem to lend themselves equally well to any subject matter, the same is not true of the corresponding *methods of investigation*. This is quite clear in the 1764 prize essay (2: 283–286) and also in the sections on method in the *Critique* where Kant compares the methods of investigation which are proper to philosophy and mathematics:

> . . . in philosophy one must not imitate mathematics in putting the definitions first, unless perhaps as a mere experiment. For since [in philosophy] they [i.e. the definitions] are analyses of given concepts, these concepts, though perhaps only still confused, come first, and the incomplete exposition precedes the complete one, so that we can often infer much from some marks that we have drawn from an as yet uncompleted analysis before we have arrived at a complete exposition, i.e., at a definition; in a word, it follows that in philosophy the definition, as distinctness made precise, must conclude rather than begin the work. (A 730 f./ B 758 f.)

Kant maintained this methodological distinction throughout the critical period. The corresponding distinction between *modes of presentation* was also maintained. We have already seen it at work in the general argumentative structures of the *Prolegomena* and the *Critique of Pure Reason*. The same can also be noticed in the modes of presentation of the *Groundwork of the Metaphysics of Morals*, which proceeds *analytically* (see 4: 392) and the *Critique of Practical Reason*, which proceeds *synthetically* (see 5: 10).

The distinction between methods of investigation and presentation of the results of an investigation was surely not overlooked in the *Critique*. Indeed, it would have been odd for Kant to maintain in the *Critique* that philosophy should proceed by *analyses* of given concepts (A 730/ B 758) and then go on to say, in the *Prolegomena* (4: 274), that he used the *synthetic* method in the *Critique*. One can only make sense of those seemingly conflicting claims by taking the distinction mentioned in the *Prolegomena* as a distinction *not* between methods of investigation, but rather between ways of presenting the results of an investigation.

We shall not further discuss here how those distinctions are to be drawn or whether they are coherent.[16] For the purposes of explaining why incongruent counterparts are absent from the first *Critique*, it suffices to note that Kant *thought* of himself as following different modes of presentation in the *Critique* and the *Prolegomena*. This is also corroborated by the fact that if we look at the structure of the arguments in the *Critique* and in the *Prolegomena*, there are apparent differences in the argumentative structure of each. As mentioned above, the *Prolegomena* begins with given cognitions, namely, the cognitions of mathematics, and seeks the general principles which make them possible. The *Critique*, on the other hand, begins with general principles and notions, and argues from them to the possibility of synthetic a priori cognitions in general.

Those differences can be seen not merely in the most general argumentative structure of the two works, but also in the strategy taken up in each particular section. Thus, in the "Transcendental Aesthetic," the arguments for the apriority and intuitiveness of space (and time) begin not with cognitions, but with general notions and distinctions. The first argument of the first section (A 23/ B 38), for example, derives the non-empirical character of space not from any given or assumed cognition, but from the impossibility of us being given certain sensations of things outside ourselves without already having the representation of space as their ground.

The argument for the a priori character of space in the *Prolegomena*, on the other hand, takes a wholly different route. It begins (§6) by assuming that we have mathematical cognitions and that they are synthetic and a priori in nature. This is not argued for but simply taken for granted at the outset. Kant then argues that those cognitions must be grounded on a priori intuitions (§7), and that in order for those a priori intuitions to be possible they must contain "nothing else except the form of sensibility" (4: 282). Hence, the possibility of a priori intuitions is inferred from there being such a thing as the form of intuition (§9). Since "space and time are intuitions upon which pure mathematics bases all its cognitions and judgments" (4: 283), it follows that space and time are a priori intuitions, given that mathematical cognitions are actual.

Similarly for other sections of the *Critique* and the *Prolegomena*: In the former, each step of the argumentation tends to be progressive, whereas in

the latter they tend to be regressive. This is, of course, only a trend, since some passages are not clearly presented in one way or the other, and there are passages which do not seem to conform to this general picture. As is well known, some sections of the *Prolegomena* were later inserted into the second edition of the first *Critique*. And more importantly, there are passages of the *Critique* which have traditionally been interpreted as following an analytic pattern of argumentation. In particular, there is the passage often referred to as the "Argument from Geometry," which appears in the section entitled "Transcendental Exposition of the Concept of Space" (B 40–42). There are good reasons to be suspicious of the traditional or standard reading of that passage, however. I shall not dwell on the point here, but merely refer the reader to a recent paper by Lisa Shabel.[17]

4. *Why Kant did not mention incongruent counterparts in the first Critique*

The various arguments from incongruent counterparts which Kant produced all follow the *analytic* mode of presentation. They all begin with a given cognition, which is taken to be evident and readily accessible to anyone: we are all capable of recognizing and identifying the form of a right human hand and telling it apart from its left counterpart, or a hand-written page and its mirror-image, and so forth. His 1768 essay showed that any two counterparts are similar and equal, but that not all counterparts are congruent. The fact that some are incongruent is plainly cognizable: a right hand does not fit into a left glove. This is not argued for but stated at the outset and taken to be evident in each of the incongruent counterparts arguments: we have the capacity to perceive differences among incongruent counterparts and to identify the form of each one, even though they are "equal and similar."[18] The arguments from then on proceed regressively.

Kant points out that all counterparts, including *incongruent* counterparts, are similar and equal. The notions of "equality" and "similarity" are technical notions here, roughly meaning sameness in magnitude and sameness in all characteristics when each object is considered in isolation. The distinction was already drawn by Leibniz and others.[19] Two objects are "equal and similar" if they have the same quantitative and qualitative characteristics. But then one would expect that any two objects that are equal and similar would be congruent, or "enclosable in the same limits." The fact that we are able to identify and recognize incongruent counterparts shows that that expectation fails.

Each of Kant's incongruent counterparts arguments then proceeds to analyze how we can apprehend differences in things that are equal and similar. In the 1768 essay, Kant argues that those differences which we apprehend must pertain to characteristics of space itself. Therefore, he

concludes, Leibniz's relational theory of space must be false. In the *1770 Dissertation* Kant says that those differences cannot be apprehended conceptually. Finally, in the *Prolegomena* and *Metaphysical Foundations*, Kant infers that space must be transcendentally ideal. In each case the conclusion is reached regressively, by asking what conditions must be satisfied for us to be able to notice the difference between similar and equal but nonetheless incongruent things.

Therefore, none of those arguments follow the synthetic mode of presentation. The arguments begin with specific cognitions which are evident and easily accessible; they then proceed regressively towards the principles which make those cognitions possible. These are the trademarks of an "analytic" argument. So here we have a reasonable explanation for why Kant left incongruent counterparts out of the *Critique*. I present it merely as a conjecture, since we cannot know for sure what, if anything, motivated Kant to leave them out. It is nonetheless a plausible conjecture, more likely to be true than those put forth by Kemp Smith and Buroker. Still, it too has its shortcomings, and I conclude this paper by pointing out four of them. First, Kant did not use the *analytic* mode of presentation in the *1770 Dissertation* and in the *Metaphysical Foundations*, but did include incongruent counterparts in both.[20] In the Preface to the *Metaphysical Foundations*, however, Kant acknowledges that he did not follow "the mathematical method with thoroughgoing rigor" (4: 478). What he means by "mathematical method" is not immediately clear in that context, but given his distinction between mathematical and philosophical methods in the *Critique*, those remarks most likely refer to the mode of presentation. Given his remarks in "Doctrine of Method" of the *Critique*, it would be unlikely that Kant would be inclined to follow mathematical method of investigation in a philosophical book. Hence, what he is saying is that he did not follow the *synthetic mode of presentation* with thoroughgoing rigor in the *Metaphysical Foundations*. Confirmation for this interpretive hypothesis is also afforded by the fact that Kant explicitly says that the argument from incongruent counterparts figures in that book as a "digression" (4: 484), something therefore out of line with the rest of the book. This mitigates the problem, though it might not completely solve it.

A second problem with the conjecture here entertained is that if one admits that the use of incongruent counterparts in the *Metaphysical Foundations* was only possible because Kant allowed for exceptions and digressions, then one might also want to reconsider Kant's use of incongruent counterparts in the *Prolegomena*, which was also somewhat of a digression.

A third problem is that it is still unclear what to say of the *1770 Dissertation*. Its overall structure suggests that Kant intended to write it in the synthetic mode, but then why do we nonetheless find an argument from incongruent counterparts in that book? Perhaps Kant just did not care to

apply the distinction too thoroughly in that book. But that, too, is still just another conjecture.

Finally, a fourth question that remains unanswered is that even if Kant did not want to use analytic arguments *from* our cognition of incongruent counterparts in the *Critique*, he could nonetheless have included synthetic arguments *to* the possibility of their cognition. The fact that he did not do so may suggest either that he thought these examples were not particularly well suited for synthetic arguments, or that he just thought that they were unimportant. The fact that Kant nowhere uses incongruent counterparts in arguments *to* the possibility of their cognition is evidence for the former alternative, but the fact that he does use other items of geometrical knowledge in the *Critique* is evidence for the latter. If the former suggestion is true, the conjecture here entertained is confirmed, but if the latter suggestion is the case, then the conjecture must be strongly mitigated.[21]

Until these questions can be answered satisfactorily, the conjecture presented in this paper must remain just that: a conjecture. Until then, the puzzle it intended to solve must, accordingly, linger on as a puzzle.[22]

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NOTES

[1] 2: 382 [references of the form "volume: page" refer to the standard pagination of the Akademie edition of Kant's works]. This passage is from Kant's most thorough discussion of incongruent counterparts, his 1768 essay "Concerning the Ultimate Ground of the Differentiation of Directions in Space" (2: 377–83) [English translation in Kant, 1992]. Examples of incongruent counterparts mentioned by Kant include pairs of human hands, human ears, snail shells, and screws. The terms 'similarity' and 'equality', with which Kant characterizes incongruent counterparts in the citation above are technical notions. They roughly mean indistinguishability when each object is considered in isolation, and sameness in magnitude, respectively. For two enlightening discussions of the history of those notions, which can be traced back to Antiquity, and further references, see Rusnock and George (1995), especially pp. 256–62, and Sutherland (2005).

[2] Hans Vaihinger points out that Kant may have learned of the existence of such objects from a book by Segner (1741) which he had in his personal library. See Vaihinger (1892), vol. 2, p. 531 n.

[3] Besides the 1768 essay, incongruent counterparts appear in the *1770 Dissertation* (2: 402 ff.), the *Prolegomena* (1783, 4: 285 f.), the *Metaphysische Anfangsgründe der Naturwissenschaft* (1786, 4: 484), and in "Was heist: Sich im Denken orientieren" (1786, 8: 131 ff.). They are also discussed in Herder's lecture notes of Kant's 1763 metaphysics course (*Metaphysik Herder*, 28.1: 15).

[4] See, for example, Torretti (1967), part one; Buroker (1981); several of the essays in Van Cleve and Frederick (1991); Rusnock and George (1995); and Walford (2001).

[5] But see Remnant (1963), Torretti (1967), Mühlhölzer (1992), Rusnock and George (1995), Walford (2001), and the present author's (2005). See also the essays in Van Cleve and Frederick (1991).

[6] *Metaphysik Herder*, 28.1: 15. Lebniz's thesis can be found in several of his own writings, for example: "But of these things one can understand simultaneous similarity and equality to be congruence" (in Gerhardt, 1965, vol. V, p. 154); "Thus, truly geometrical analysis does not only see equalities and proportions, the latter of which are in fact reduced to the former, but also similarities, and to the conjunctions of equality and similarity one must apply congruence" (ibid., p. 179). See also Wolff, 1751, p. 1179. The notions of similarity and equality can be traced back to antiquity, but Leibniz seems to have been the first to formulate a general notion of "similarity." Hence, the general thesis that all equal and similar things are congruent seems to be originally Leibniz's. For discussion, see Rusnock and George (1995), Sutherland (2005), and the present author's (2005).

[7] *Inquiry Concerning the Distinctness of the Principles of Natural Theology and Morality* (in Kant, 1992), 2: 275–301, relevant passages in 2: 276 f.

[8] For a further discussion, see Jong (1995), and the present author's (2005).

[9] For discussion, see the essays mentioned in endnotes 4 and 5 above.

[10] On Kant's early uses of Leibniz's relational view of space, and his attempts to reconcile it with Newtonian science, see Torretti (1967, part I), and Friedman (1992, introduction).

[11] *De Mundi Sensibilis atque Intelligibilis Forma et Principiis*, 2: 385–419 (English translation in Kant, 1992).

[12] (English translation taken from Kant, 2002.) What Kant must have had in mind here were instances of *scalene* spherical triangles. All equilateral and some isosceles spherical triangles have *congruent* counterparts, but all scalene spherical triangles have *incongruent* counterparts.

[13] Kemp Smith, 1923, pp. 161 ff.

[14] See KrV, A 13/ B 27; *The Jäsche Logic*, 9: 19 f. and 9: 148 ff. (English translation in Kant, 2004). For discussion and further references, see Jong (1995) and Caimi (1996). The issue is of course connected to a methodological distinction which dates back to the Ancient Greeks. See, for example, Pappus (in Thomas, 1941), pp. 596–599. For discussion and further references, see Hintikka (1973), and Jong (1997).

[15] "Analytic method is more appropriate for the end of popularity, synthetic method for the end of scientific and systematic preparation of cognition." (*The Jäsche Logic*, §155 n., 9: 149; English translation in Kant, 2004) "Method can be critical, scholastic, also popular. In scholastic method the exposition is composed methodically[,] i.e., where the parts of the method, and meticulousness of observation, shine forth. This method reigns in all the sciences. Popular method does not have the purpose of furthering science but instead of furthering interest (. . .). As far as the popular is concerned, one has to attend to the fact that popular exposition and popular method are not the same. The exposition can be popular[,] but not the method. For example, Gottsched's compendium is popular[,] but its method is scholastic. The French all have popular exposition. (. . .) Scientific method is divided into synthetic and analytic method. With synthetic method one begins with principles of reason and proceeds toward things that rest on principles[,] with analytic method one proceeds toward principles from things that rest on principles. (. . .) Analytic method is always combined with popularity, for one gets used to abstract cognitions when one ascends to principles rather than having to begin with them. Synthetic method is the most perfect of all (. . .). Analytic method is also a means of discovery and of exposition, in that I speak popularity. The true method of exposition is synthetic, however, for even if I have thought the thing analytically, the systematic method is what first makes it a system" (*The Hechsel Logic*, 24: 115 ff.; English translation in Kant, 2004).

[16] But see Jong (1995) and Caimi (1996).

[17] Shabel, 2004.

[18] This is particularly evident in Kant's 1768 essay, the first half of which is almost exclusively devoted to a rather detailed presentation of plainly available facts and cognitions we are all capable of, having to do with orientations and directions in space.

[19] See Loemker, 1956, vol. I, p. 392. For discussion, see Rusnock and George (1995) and Sutherland (2005).

[20] In "Was heisst: Sich im Denken orientieren?" Kant also discusses incongruent counterparts, but that essay was clearly written according to the *analytic mode* of presentation.

[21] This point was raised by Bradford Cokelet and by an anonymous reviewer.

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