Structures of Repetitive Controllers Based on GDSC with Feedforward Action

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***Abstract*—This work presents a vector repetitive control scheme for three-phase systems, based on the generalized delayed signal cancelation transform. Three proposed control structures are presented, allowing to regulate a family of harmonic components ($nk + m$, $k \in \mathbb{Z}$), where the negative harmonics indicate negative-sequence components. Differently from most repetitive controllers, distinct negative- and positive-sequence components can be regulated. A stability evaluation is presented. Experimental results are shown in order to validate the proposed control scheme and demonstrate its good performance when applied to a three-phase active power filter.**

***Index Terms*—Generalized delayed signal cancelation (GDSC), repetitive control, harmonics compensation, vector control.**

I. Introduction

Repetitive controllers (RC), as originally proposed [1], have infinite gain for a selected frequency and all of its harmonic components. Its control structure has a periodic signal generator, enabling to eliminate the steady-state error for periodic reference signals, according to the internal model principle [2]. Once the original RC compensates all harmonic components makes necessary to store the samples of the measured signals during the last fundamental period. As a consequence, a response time of at least one fundamental cycle is expected.

RCs with the ability of regulating only the harmonic components in families of the type ($4k \pm 1$, $k \in \mathbb{N}$) (all odd components) [3], ($6k \pm 1$, $k \in \mathbb{N}$) [4] or ($nk \pm m$, $k \in \mathbb{N}$) [5] were proposed. Their main advantage is the smaller number of samples to be stored, resulting in a faster response time.

However, for three-phase systems that use a space vector as reference signal, the harmonic spectrum is evaluated so that positive and negative frequencies represent positive- and negative- sequence harmonic components, respectively. By using this concept, complex RCs can be applied to the space vector error in order to control the harmonic components in the familiy ($nk + m$, $k \in \mathbb{Z}$), as the one presented in [6]. This class of RC is here refered as space vector repetitive controller (SV-RC).

Many authors have researched about RC-based solutions applied to harmonic cancellation, e. g., [4] [7]. Considering this application, SV-RCs present some advantages when compared to $nk \pm m$ RCs. In fact, SV-RCs provide a faster transient performance and require less memory cells. Also, the dual-input/dual-output control system (in the $\alpha\beta$ reference frame), can be simplified into one single-input/single-output system expressed with complex-vector notation [6].

The generalized delayed signal cancelation (GDSC) is a complex mathematical transform applied to space vectors of three-phase signals [8]. It was proposed for eliminating harmonic components of unbalanced and distorted signals and being used as a pre-filter for three-phase-PLLs. In this paper, a SV-RC structure based on the GDSC transform is proposed for three-phase systems. The proposed controller has infinite gain for a family of harmonic components ($nk + m$, $k \in \mathbb{Z}$), where the negative values correspond to negative-sequence components. Zero steady-state error can then be ensured for different positive- and negative-sequence components, allowing a reduction in the number of samples of measured quantities to be stored and also a faster response.

II. The GDSC Transform

The GDSC transform uses the current and delayed space vectors of a three-phase signal, being defined as [8] [9]

$$\vec{f}_{gdsc}[i] = \vec{a}\left\{\vec{s}_{\alpha\beta}[i] + e^{j\theta_r}\vec{s}_{\alpha\beta}[i - i_d]\right\}, \quad (1)$$

where i is the current sample, i_d defines the delay in number of samples, θ_r is an angle by which the delayed space vector $\vec{s}_{\alpha\beta}[i - i_d]$ is rotated and $\vec{a}$ is a constant complex gain.

Through this transformation it is possible to cancel the vector harmonic components in the set ($h = nk + m, k \in \mathbb{Z}$) of the original signal. Thus, let m be one harmonic component to be eliminated and n the periodicity that defines the other components to be canceled, the parameters i_d and θ_r are determined by making the GDSC complex gain equal to zero:

$$\vec{G}^{(h=nk+m)}_{gdsc} = \vec{a}(1 + e^{j\theta_r}e^{-j(nk+m)\frac{2\pi}{N}i_d}) = 0, \quad (2)$$

where N is the number of samples per fundamental period. Consequently, the parameters i_d and θ_r can be calculated from

$$\begin{cases} i_d = \dfrac{N}{n} \\ \theta_r = \dfrac{m}{n}2\pi + \pi \end{cases}. \quad (3)$$

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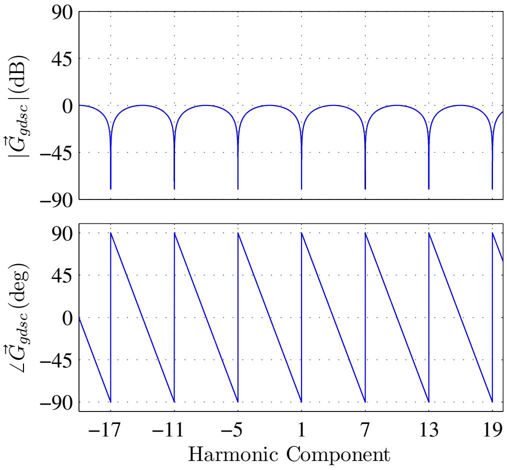

Fig. 1. Bode diagram of the GDSC transform for canceling the harmonic components in $(6k+1, k \in \mathbb{Z})$.

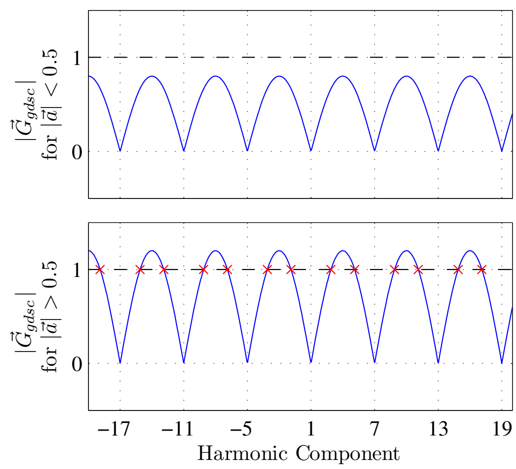

Fig. 3. Controller frequency response for $|\vec{a}| < 0.5$ (top) and for $|\vec{a}| > 0.5$ (bottom).

The complex gain $\vec{a}$ is determined for imposing unit gain to a desired harmonic component.

The frequency response of the GDSC transform designed for eliminating the harmonic components in the family $(6k+1, k \in \mathbb{Z})$ is shown in Fig. 1. In this example the GDSC parameters are $i_d = N/6$, $\theta_r = 4\pi/3$ and $\vec{a} = 0.5$.

III. Vector RC Based on GDSC

A general way to implement a SV-RC is presented in Fig. 2. In this structure the GDSC transform is used in the direct path of a loop with positive feedback, following the idea proposed in [10] for a controller based on the space vector Fourier transform (SVFT). A feedforward action is added to the controller through a second direct loop with gain "b", allowing to select one of the configurations presented below. The transfer function of the controller having this structure is

$$\vec{C}_{gdsc}(z) = b + \frac{\vec{G}_{gdsc}(z)}{1-\vec{G}_{gdsc}(z)} = \frac{b+\vec{G}_{gdsc}(z)(1-b)}{1-\vec{G}_{gdsc}(z)}. \quad (4)$$

It is important to note that this structure presents infinite gain in the frequencies for which the GDSC transform gain is equal to one. As shown in the previous section, the parameter $\vec{a}$ can be used to define the gain of the GDSC transform at some desired frequency. A convenient value is $\vec{a} = 0.5$, since in this case the transform presents unit gain for frequencies in the family $(nk+m,\ k \in \mathbb{Z})$.

If $|\vec{a}| > 0.5$ there will be two frequencies in the chosen periodicity for which the gain is equal to one. This makes the controller to lose the ability to regulate harmonic components in the family $(nk+m, k \in \mathbb{Z})$. On the other hand, if $|\vec{a}| < 0.5$ the controller characteristics are lost, since the GDSC transform would not present unit gain for any harmonic component. Fig. 3 exhibits the absolute value of the GDSC gains when designed to cancel harmonic components in the family $(6k+1, k \in \mathbb{Z})$ using different values of $|\vec{a}|$, in order to illustrate the two situations mentioned.

Three possible implementation configurations for the proposed SV-RC, all of them based on the structure shown in Fig. 2, are presented as follows.

A. Configuration 1 ($C1$): $b = 0$

Making $b = 0$ cancels the effect of the feedforward action. In this case the transfer function of the proposed controller, shown in Fig. 4, becomes

$$\vec{C}_{gdsc-C1}(z) = \frac{\vec{G}_{gdsc}(z)}{1-\vec{G}_{gdsc}(z)} = \frac{\vec{a}(1+e^{j\theta_r}z^{-i_d})}{1-\vec{a}(1+e^{j\theta_r}z^{-i_d})}. \quad (5)$$

Considering $\vec{a} = 0.5$, the transfer function changes to

$$\vec{C}_{gdsc-C1}(z) = \frac{\vec{U}_{\alpha\beta}(z)}{\vec{E}_{\alpha\beta}(z)} = \frac{1+e^{j\theta_r}z^{-i_d}}{1-e^{j\theta_r}z^{-i_d}}, \quad (6)$$

whose block diagram is presented in Fig. 5.

This SV-RC is similar to the one proposed in [11] (making the parameter $a(s) = 0.5$). The main difference is that instead of using phase quantities (a, b, and c) as inputs, it uses a space vector. An equivalent structure was proposed in [6].

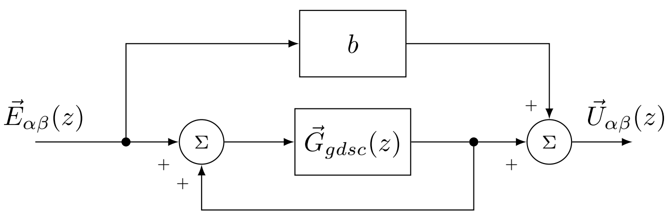

Fig. 2. Block diagram of the SV-RC based on GDSC.

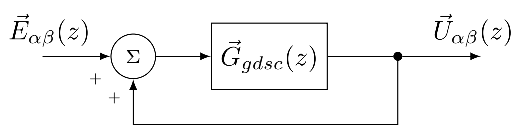

Fig. 4. GDSC based controller for $b = 0$ ($C1$).

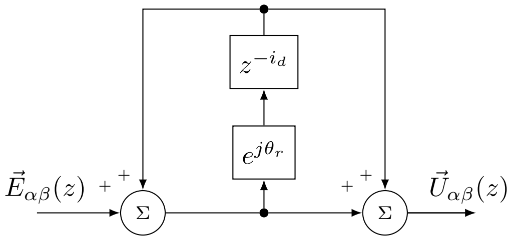

Fig. 5. Detailed block diagram of $C1$ with $\vec{a} = 0.5$.

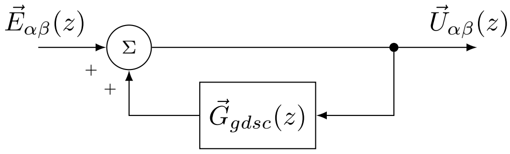

Fig. 6. GDSC based controller for $b = 1$ ($C2$).

B. Configuration 2 ($C2$): $b = 1$

The parameter $b = 1$ can be selected for adding the error to the control action. In this case the controller transfer function is

$$\vec{C}_{gdsc-C2}(z) = \frac{1}{1 - \vec{G}_{gdsc}(z)} = \frac{1}{1 - \vec{a}(1 + e^{j\theta_r} z^{-i_d})}, \quad (7)$$

which is equivalent to the block diagram of Fig. 6. By making $\vec{a} = 0.5$, the transfer function of the controller becomes

$$\vec{C}_{gdsc-C2}(z) = \frac{\vec{U}_{\alpha\beta}(z)}{\vec{E}_{\alpha\beta}(z)} = \frac{2}{1 - e^{j\theta_r} z^{-i_d}}. \quad (8)$$

C. Configuration 3 ($C3$): $b = -1$

By making $b = -1$ the controller transfer function is computed by

$$\vec{C}_{gdsc-C3}(z) = \frac{-1 + 2\vec{G}_{gdsc}(z)}{1 - \vec{G}_{gdsc}(z)} = \frac{-1 + 2\vec{a}(1 + e^{j\theta_r} z^{-i_d})}{1 - \vec{a}(1 + e^{j\theta_r} z^{-i_d})}. \quad (9)$$

Thus, for $\vec{a} = 0.5$, it is possible to obtain a structure similar to the RC proposed in [11] with the parameter $a(s) = 0$. The transfer function of the resulting controller is

$$\vec{C}_{gdsc-C3}(z) = \frac{\vec{U}_{\alpha\beta}(z)}{\vec{E}_{\alpha\beta}(z)} = \frac{2e^{j\theta_r} z^{-i_d}}{1 - e^{j\theta_r} z^{-i_d}}, \quad (10)$$

which can be represented through the block diagram of Fig. 7.

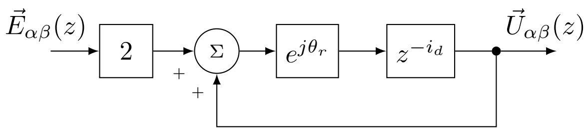

Fig. 7. Detailed block diagram of $C3$ with $\vec{a} = 0.5$.

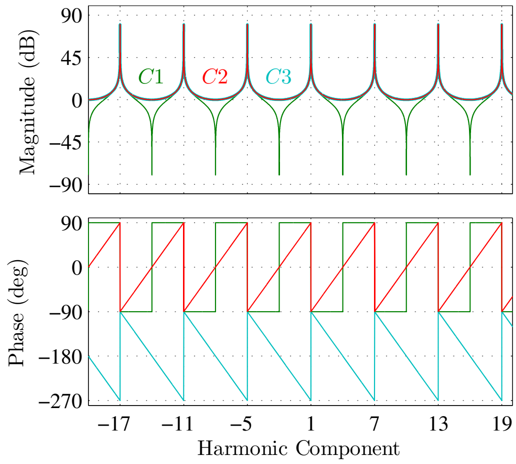

Fig. 8. Frequency responses of the three proposed control structures designed for $(6k + 1, k \in \mathbb{Z})$.

IV. Comparison of the Proposed Structures

The frequency responses of the three configurations of the proposed SV-RC are exhibited in Fig. 8.

The stability of a discrete-time control system is frequently evaluated by observing the position of its closed-loop poles, which must be inside the unit circle. For the system represented by the block diagram of Fig. 9, whose closed-loop transfer function is

$$\frac{\vec{Y}^d(z)}{\vec{R}^d(z)} = \frac{\vec{C}_{gdsc}(z)\vec{G}_p(z)}{1 + \vec{C}_{gdsc}(z)[\vec{G}_p\vec{H}](z)}, \quad (11)$$

in which its poles are calculated from

$$1 + \vec{C}_{gdsc}(z)[\vec{G}_p\vec{H}](z) = 0. \quad (12)$$

Block $\vec{G}_p(z)$ represents the discretized vector plant, while $[\vec{G}_p\vec{H}](z)$ represents the plant and the sensors. The sampled signals, evaluated in discrete-time, are represented using the superscript "d".

The proposed SV-RC based on GDSC has i_d poles distributed on the unit circle, separated by an angle $\theta = 2\pi/i_d$. The poles and zeros for $C3$ are exemplified in Fig. 10.

Due to the controller complexity, it becomes difficult to compare the stability properties of the configurations of the proposed controller using the poles and zeros location on the complex plane. On the other hand, the stability analysis presented in [11] can be extended to the SV-RCs, allowing

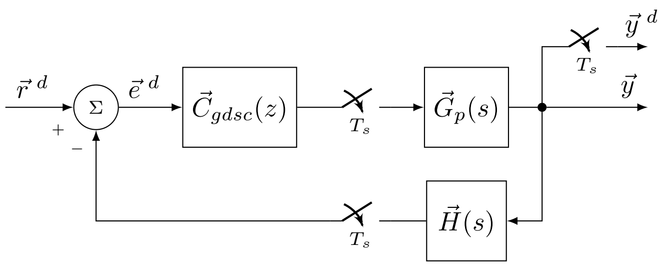

Fig. 9. Control system with SV-RC using samplers.

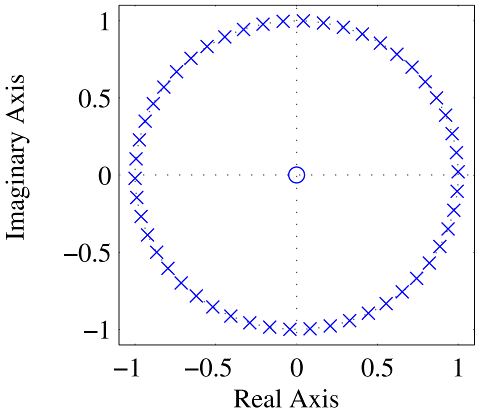

Fig. 10. Poles and zeros of $C3$, for $i_d = 50$.

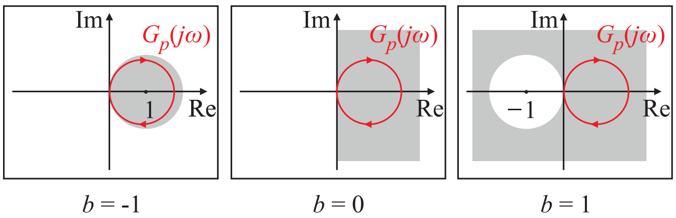

Fig. 11. Stability domains for systems with SV-RC, based on [11].

an initial comparison. Thus, the controlled plant must be restricted to Haras stability domains, i. e., its Nyquist diagram must be in the shaded areas of Fig. 11, which vary according to the configuration used. As it can be seen, configuration 2 has the largest stability domain.

The origin of the complex plane is on the border of the stability domain, regardless the configuration used. This means that the controllers application is restricted to plants with relative degree equal to 0, which have equal number of poles and zeros. In order to enlarge the stability domains, a low-pass filter (LPF) ,generally a finite-impulse response (FIR) filter, is frequently used along with the controller delay block [3] [11].

V. Experimental Results

In order to validate the proposed controller and to evaluate its performance, a three-phase active power filter (APF) for compensating the harmonic currents of a nonlinear load was used. Fig. 12 presents a schematic diagram of the prototype. The proposed configuration with $b = 1$, which presents the biggest stability domain ($C2$) was implemented. The parameters of the prototype have the values shown in Table I.

The results were compared with those obtained using the controller $6k \pm 1$ RC as proposed in [12], using a dSPACE platform.

Fig. 13 presents plots of the grid currents before and after enabling the APF operation, using the proposed controller. The corresponding results obtained using the RC controller proposed in [12] are presented in Fig. 14, which parameters were tuned in order to achieve the same phase margin. In both cases, the APF operation is enabled at $t = 0$s. Both Fig. 13 and Fig. 14 are plotted from $t = -0.01$s to $t = 0.06$s in order to display the controller transient.

The levels of the currents distortion were evaluated using the concept of vector total harmonic distortion ($VTHD$) [8]. The $VTHD$ was computed considering the harmonic components from $h = -50$ until $h = 50$, where the minus signal indicates negative-sequence. In Figs. 13 and 14, $VTHD_1$ represents the $VTHD$ of the three-phase grid currents before the APF operation, while $VTHD_2$ indicates the $VTHD$ of the same currents when the APF is operating under steady state condition. As it can be observed, the proposed control scheme led to a better control steady-state performance, when compared to the strategy proposed in [12].

VI. Conclusion

This paper proposes the use of the GDSC for implementing a space vector repetitive controller, able to regulate a set of harmonic components selected by the designer, ensuring zero steady-state errors. An important characteristic of the presented controller is its frequency and sequence selectivity, being able to compensate different positive- and negative-sequence components. Three distinct configurations were presented and their stability characteristics were evaluated, so that configuration 2 presented the largest stability domain. This configuration was implemented and used for an APF control. For comparison purposes, the repetitive control proposed in [12] was also implemented. The main advantages of the proposed scheme are the expected faster response due to the smaller delay used for its implementation and the corresponding lower amount of memory required. In the experimental evaluation performed, the proposed scheme led to better steady-state harmonic compensation.

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TABLE I
Parameters of the experimental prototype.

Prototype									Controller	
$V_{g(line)}$	L_g	R_g	L_l	L_f	R_f	R_{load}	V_{dc}	f_s	K_{rc}	N
380 V_{rms}	186.17 μH	31.7 mΩ	1.4830 mH	2.5635 mH	307.5 mΩ	48.4 Ω	600 V	17.28 kHz	0.030	288

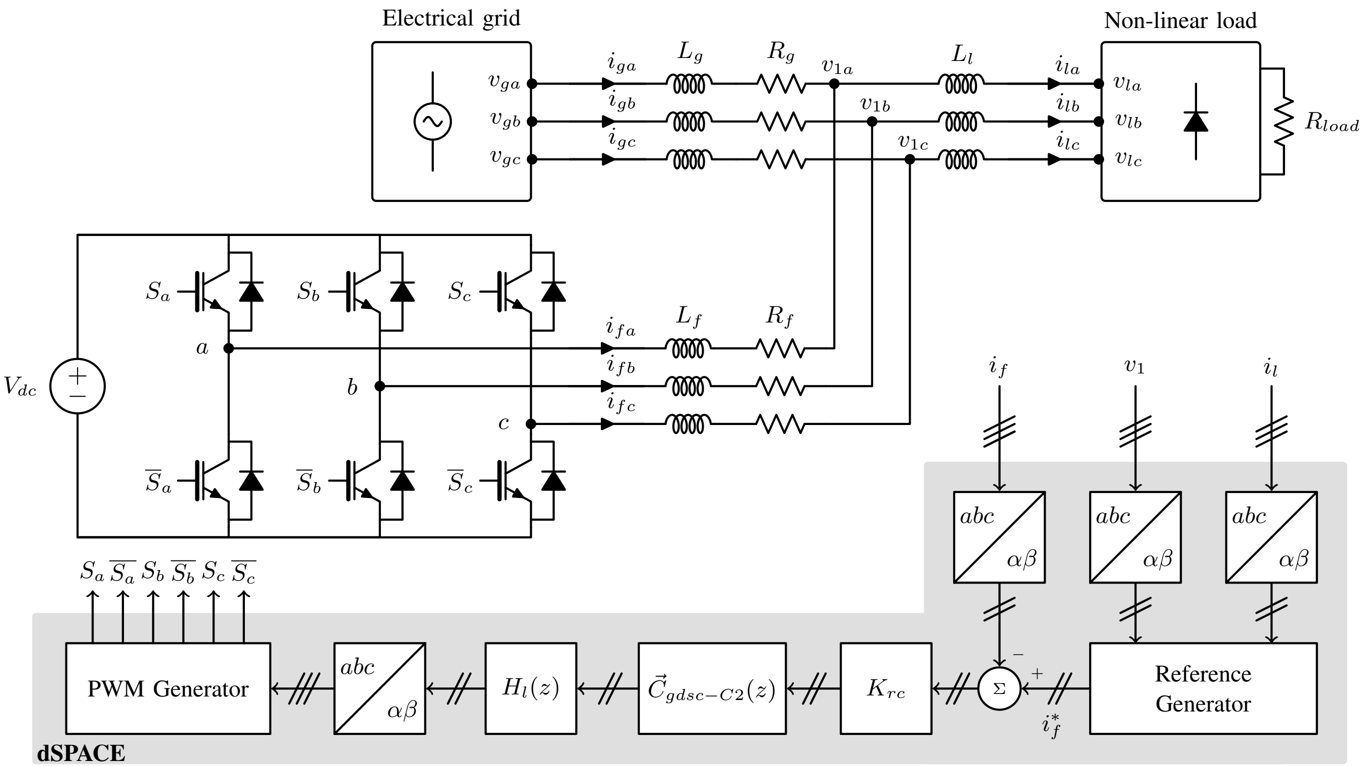

Fig. 12. Complete diagram of the system used for validating the proposed controller.

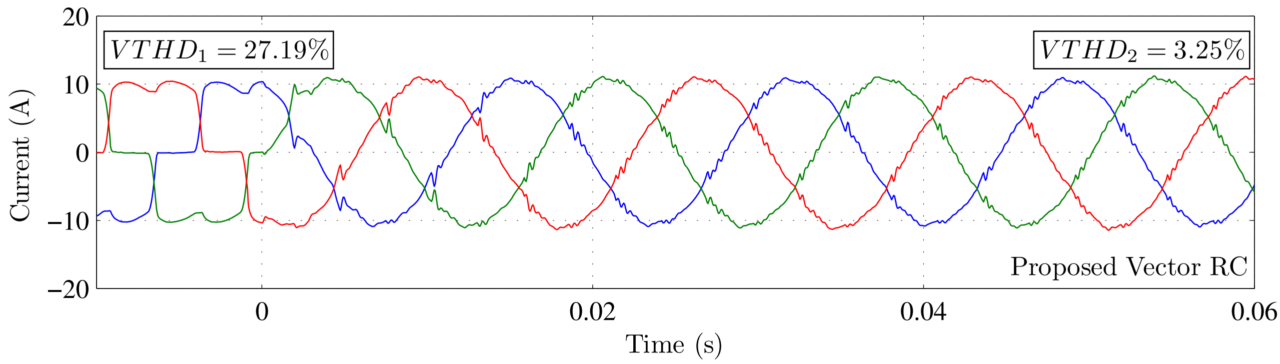

Fig. 13. Experimental results using $C2$ for regulating the harmonic components in $(6k+1,\ k \in \mathbb{Z})$. Source currents before and after APF operation.

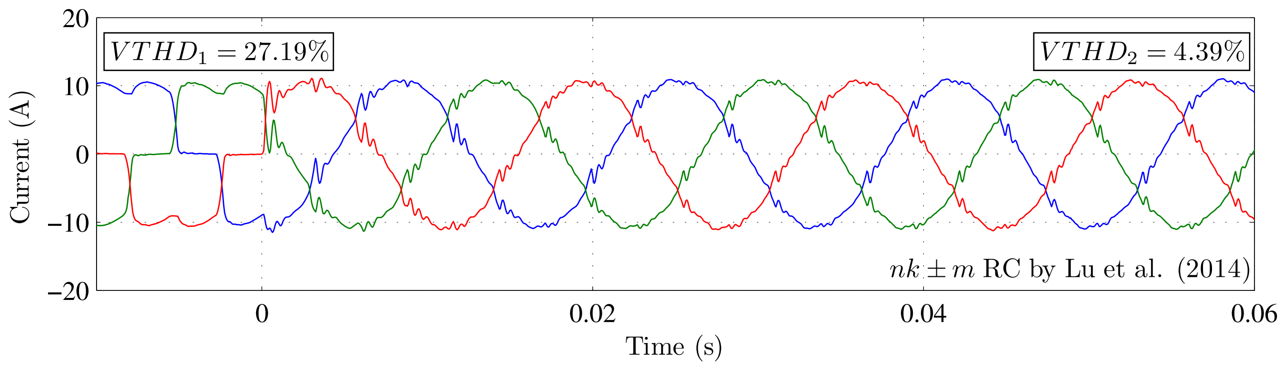

Fig. 14. Experimental results for $6k \pm 1$ RC proposed in [12]. Source currents before and after APF operation.

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