UNIVERSIDADE FEDERAL SANTA MARIA
DEPARTAMENTO DE FÍSICA
Programa de Pós-Graduação em Física
List 4
Segundo semestre/2015
FSC 810 Mecânica Estatística

1.3 A number is chosen at random between 0 and 1. What is the probability that exactly 5 of its first 10 decimal places consist of digits less than 5?

1.4 A drunk starts out from a lamppost in the middle of a street, taking steps of equal length either to the right or to the left with equal probability. What is the probability that the man will again be at the lamppost after taking N steps

(a) if N is even?
(b) if N is odd?

1.6 Consider the random walk problem with $p = q$ and let $m = n_1 - n_2$ denote the net displacement to the right. After a total of N steps, calculate the following mean values: $\overline{m}$, $\overline{m^2}$, $\overline{m^3}$, and $\overline{m^4}$.

1.8 Two drunks start out together at the origin, each having equal probability of making a step to the left or right along the x axis. Find the probability that they meet again after N steps. It is to be understood that the men make their steps simultaneously. (It may be helpful to consider their relative motion.)

1.9 The probability $W(n)$ that an event characterized by a probability p occurs n times in N trials was shown to be given by the binomial distribution

$$W(n) = \frac{N!}{n!(N-n)!} p^n (1-p)^{N-n} \tag{1}$$

Consider a situation where the probability p is small ($p \ll 1$) and where one is interested in the case $n \ll N$. (Note that if N is large, $W(n)$ becomes very small if $n \to N$ because of the smallness of the factor p^n when $p \ll 1$. Hence $W(n)$ is indeed only appreciable when $n \ll N$.) Several approximations can then be made to reduce (1) to simpler form.

(a) Using the result $\ln(1-p) \approx -p$, show that $(1-p)^{N-n} \approx e^{-Np}$.
(b) Show that $N!/(N-n)! \approx N^n$.
(c) Hence show that (1) reduces to

$$W(n) = \frac{\lambda^n}{n!} e^{-\lambda} \tag{2}$$

where $\lambda = Np$ is the mean number of events. The distribution (2) is called the "Poisson distribution."

1.10 Consider the Poisson distribution of the preceding problem.

(a) Show that it is properly normalized in the sense that $\sum_{n=0}^{N} W_n = 1$.

(The sum can be extended to infinity to an excellent approximation, since W_n is negligibly small when $n \gtrsim N$.)

(b) Use the Poisson distribution to calculate $\overline{n}$.
(c) Use the Poisson distribution to calculate $\overline{(\Delta n)^2} = \overline{(n - \overline{n})^2}$.

1.14 A penny is tossed 400 times. Find the probability of getting 215 heads. (Suggestion: use the Gaussian approximation.)

1.16 Consider a gas of N_0 noninteracting molecules enclosed in a container of volume V_0. Focus attention on any subvolume V of this container and denote by N the number of molecules located within this subvolume. Each molecule is equally likely to be located anywhere within the container; hence the probability that a given molecule is located within the subvolume V is simply equal to V/V_0.

(a) What is the mean number $\bar{N}$ of molecules located within V? Express your answer in terms N_0, V_0, and V.

(b) Find the relative dispersion $\overline{(N - \bar{N})^2}/\bar{N}^2$ in the number of molecules located within V. Express your answer in terms of $\bar{N}$, V, and V_0.

(c) What does the answer to part (b) become when $V \ll V_0$?

(d) What value should the dispersion $\overline{(N - \bar{N})^2}$ assume when $V \to V_0$? Does the answer to part (b) agree with this expectation?

1.17 Suppose that in the preceding problem the volume V under consideration is such that $0 \ll V/V_0 \ll 1$. What is the probability that the number of molecules in this volume is between N and $N + dN$?