Inverse Freezing in the Fermionic van Hemmen Spin Glass Model

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Abstract

The inverse freezing (IF) transition is studied with a quantum fermionic van Hemmen spin glass model. The disorder is treated without the use of replica method, in which an exact mean field solution is obtained for two different types of quenched disorders: the bimodal and the gaussian ones. The IF is then observed for certain range of chemical potential when the gaussian distribution is adopted. However, IF is destroyed by the quantum fluctuations. Particularly, the results suggest that the nontrivial SG free energy landscape, represented by strong disordered SG models, is not a necessary condition to generate a spontaneous IF.

Introduction

Inverse transitions (IT) are a class of reversible phase transitions in which the usually ordered phase appears at higher temperature than the disordered one. In this case, there is an inversion of the entropic contents between the ordered and disordered phases. Some magnetic models have been proposed to investigate the features of the IT [1]. For example, the Blume-Capel (BC) model [2], the Ghatak-Sherrington (GS) model [3] and in infinite-range fermionic Ising spin glass (FISG) models with a transverse magnetic field Γ [4, 5, 6] in which a first-order IF can be found in phase diagrams of the temperature versus the chemical potential. The chemical potential introduces statistical charge fluctuation, what controls the site occupation. In the present work the fermionic van Hemmen spin glass model in the presence of magnetic transverse field Γ is used to study the inverse freezing (IF), which is an IT characterized by a transition between a paramagnetic phase at low temperatures and a spin glass phase at high temperatures [7]. In this model, the spin operators are written as a bilinear combination of fermionic operators, which allows the analysis of the interplay between charge and spin fluctuations in the presence of a Γ field. The problem is expressed in the fermionic path integral formalism and the disorder is treated without the use of replica method. The thermodynamic potential is obtained with two different types of quenched disorders: bimodal (discrete) and gaussian (continuous).

Model

The fermionic van Hemmen (FvH) model with a magnetic transverse field Γ is described by

$$H = -2\frac{J_0}{N}\sum_{i\neq j}\hat{S}_i^z\hat{S}_j^z - 2\sum_{i\neq j}J_{ij}\hat{S}_i^z\hat{S}_j^z - 2\Gamma\sum_i \hat{S}_i^x \qquad (1)$$

where the sums are over the N sites. The spin operators are defined in terms of fermion creation ($c_{i\sigma}^{\dagger}$) and annihilation ($c_{i\sigma}$) operators. J_0 represents a direct ferromagnetic coupling and J_{ij} is the desordered coupling given by $J_{ij} = J/N[\xi_i\eta_i + \xi_j\eta_j]$. ξ_i and η_i are independent random variables subject to a certain probability distribution, bimodal and gaussian. Both distributions introduces frustration in the problem, but the continuous one presents a larger number of different values of ferromagnetic and antiferromagnetic interactions J_{ij}, that can generate a nontrivial frustration. The partition function is obtained within the gran canonical ensemble by using the Lagrangian path formalism. The gran canonical potential per site $\Omega = -\frac{1}{N\beta}\langle\langle \ln Z\rangle\rangle$ is given by

$$\beta\Omega = \frac{\beta J_0}{2}m^2 + \beta J q^2 - \beta\mu - \langle\langle \ln 2K(\xi,\eta)\rangle\rangle \qquad (2)$$

which $K(\xi,\eta) = \cosh(\beta\mu) + \cosh\beta\sqrt{[J_0 m + J(\xi+\eta)q]^2 + \Gamma^2}$.

The order parameters spin glass and magnetization are obtained

$$q = \frac{1}{2}\left\langle\left\langle (\xi+\eta)\frac{h_i}{\sqrt{\Delta}}\frac{\text{senh}(\beta\sqrt{\Delta})}{\cosh(\beta\mu)+\cosh(\beta\sqrt{\Delta})}\right\rangle\right\rangle \qquad (3)$$

$$m = \left\langle\left\langle \frac{h_i}{\sqrt{\Delta}}\frac{\text{senh}(\beta\sqrt{\Delta})}{\cosh(\beta\mu)+\cosh(\beta\sqrt{\Delta})}\right\rangle\right\rangle \qquad (4)$$

with $\Delta = h_i^2 + \Gamma^2$ and $h_i = [J_0 m + J(\xi+\eta)q]$.

The notation $\langle\langle ...\rangle\rangle$ represents the average over the random variables ξ and η that follow either the bimodal distribution or the gaussian one, respectively expressed by

$$P(\xi,\eta) = \frac{1}{4}[\delta(\xi-1)+\delta(\xi+1)][\delta(\eta-1)+\delta(\eta+1)] \qquad (5)$$

$$P(\xi,\eta) = \frac{1}{\sqrt{2\pi}}\exp\left(\frac{-\xi^2}{2}\right)\frac{1}{\sqrt{2\pi}}\exp\left(\frac{-\eta^2}{2}\right) \qquad (6)$$

Results

The analitical results are obtained by using the partition function evaluated in the fermionic path integral formalism. Phase diagrams T/J versus μ/J are build for the two types of disorder and several values of the Γ field. They show a second-order transition from the paramagnetic phase to the spin glass one when μ is small. The increases of μ decreases the freezing tempetarure T_f until a tricritical point, after which the transition becomes first-order. The IF is then observed for a certain range of μ only when the gaussian distribution is adopted. However, the IF is destroyed by quantum fluctuations introduced by Γ.

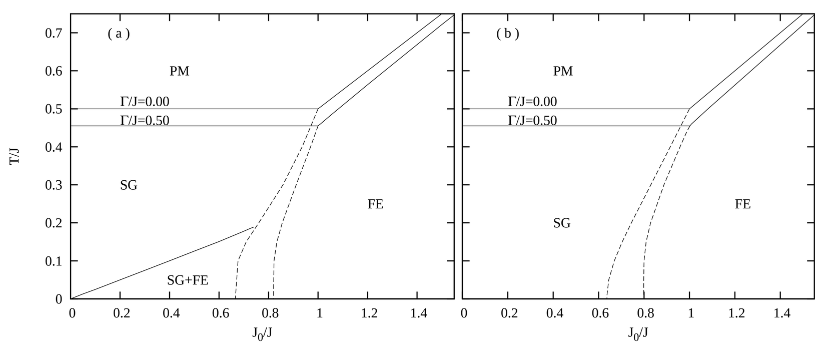

Figure 1: Phase diagrams T/J versus J_0/J for $\mu/J = 0$ with two values of Γ/J : 0 and 0.5. The solid and dashed lines represent second and first order transitions, respectively. Panels (a) and (b) show phase diagrams for the bimodal and gaussian distributions respectively.

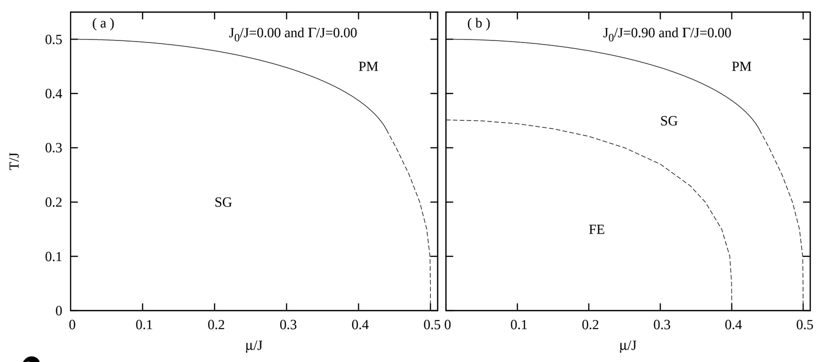

Figure 2: Phase diagrams T/J versus μ/J for the bimodal distribution, $\Gamma/J = 0$ and two values of J_0. Results for $J_0/J = 0.00$ and $J_0/J = 0.90$ are presented in panels (a) and (b) respectively. Here the same convention lines as in Fig.?? is used.

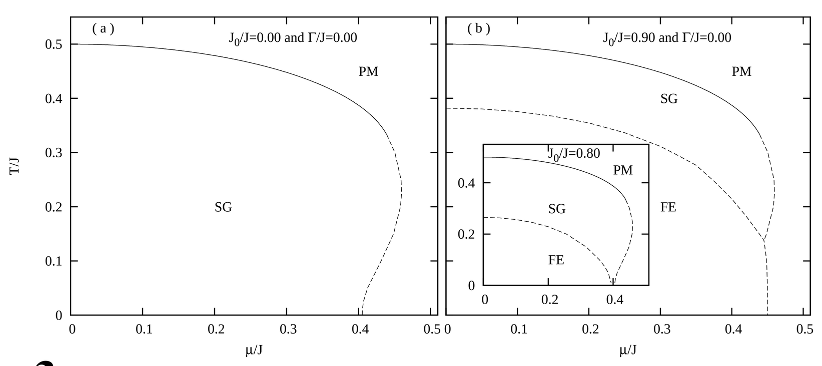

Figure 3: Phase diagrams T/J versus μ/J for $\Gamma/J = 0$ and gaussian distribution. Panels (a) and (b) exhibit results for $J_0/J = 0.0$ and $J_0/J = 0.9$, respectively. The inset in panel (b) presents results for $J_0/J = 0.80$. The solid lines represent second-order transitions while the dashed lines represent first-order transitions.

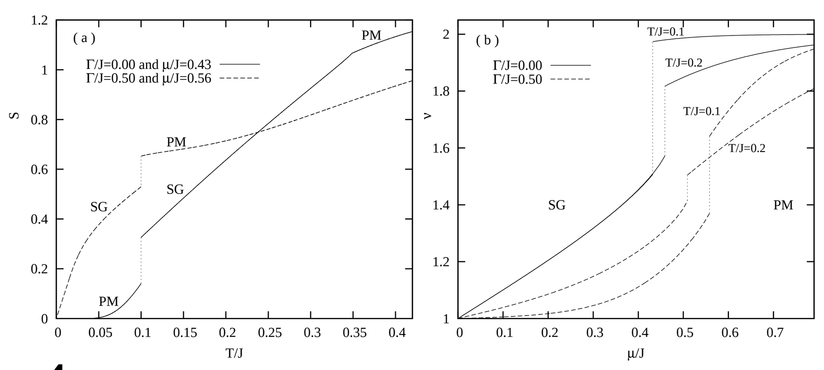

Figure 4: (a) Entropy as a function of temperature for the gaussian distribution with $J_0/J = 0$ and $\Gamma/J = 0.50$. (b)ν versus μ/J for two isotherms, $J_0/J = 0$, $\Gamma/J = 0.00$ and $\Gamma/J = 0.5$.

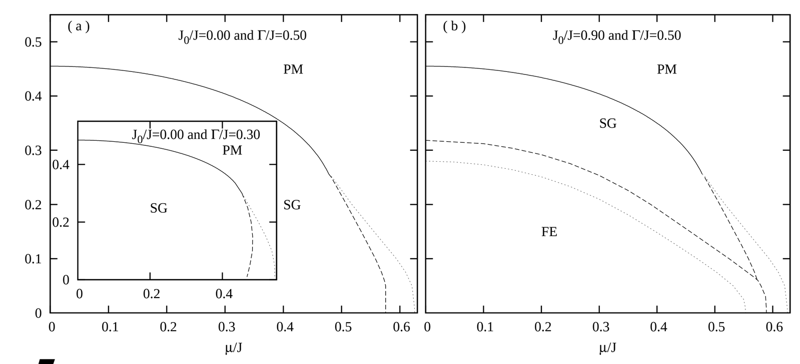

Figure 5: Phase diagrams T/J versus μ/J when $\Gamma/J = 0.5$. Panels (a) and (b) exhibit results for $J_0/J = 00$ and $J_0/J = 0.90$, respectively. The dotted and dashed lines represent first order transitions for the bimodal and gaussian distributions respectively. The inset in panels (a) presents results for $J_0/J = 0.00$ and $\Gamma/J = 0.3$

Conclusions

The present work has studied the IF phenomenon by adopting a fermionic formulation for the van Hemmen SG model in the presence of a Γ field. This quantum SG problem allows an analytical treatment, in which the partition function is evaluated in the fermionic path integral formalism. An exact mean-field SG solution is then obtained without using the replica method, in which the grand canonical potential is analyzed for two different disorder interactions: one given by the bimodal distribution and the other one by the gaussian distribution. The results suggest that the combined effect of frustration and magnetic dilution (introduced by μ) are relevant to generate spontaneously Inverse Freezing, while nontrivial Spin Glass free energy landscape represented by strong disordered SG models (where it is used the of replicas method) is not a necessary condition.

Acknowledgment

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