Laboratório de Teoria da Matéria Condensada

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TOPOLOGICAL FEATURES OF CRITICALITY IN ISING FRACTAL LATTICES

INTRODUCTION

The Ising model[1][2] can be used to study the ferromagnetic behavior that some materials present. It consists in spins that can assume two possible configurations in only one direction. These configurations are regulated by temperature and magnetic field. The critical temperature (T_c) sets where the ferromagnetism state turns to paragmagnetism. Fractals lattices[3][4][5][6] expand the perspective of how this magnetism behaves, offering topological features such as fractal dimension (D_f), lacunarity (L) and average coordination number (z). With this features, we present the role of geometric properties in the critical behavior of magnetism.

METHODS

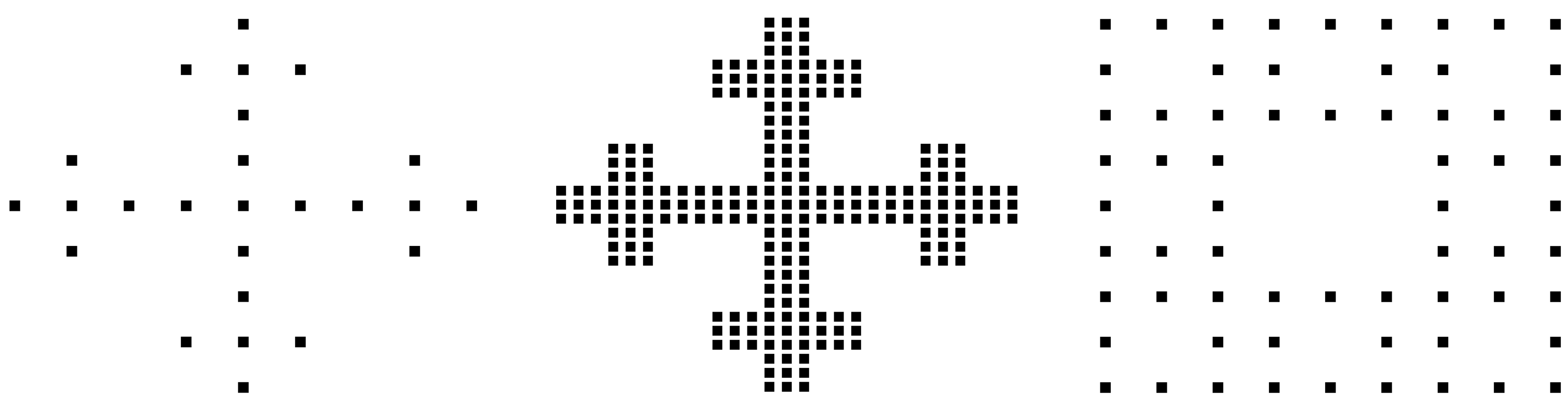


Image 1 - Some of the studied fractal lattices.

We simulated several fractal lattices to obtain their critical temperatures. The Ising Hamiltonian:

$$H = -J \sum_{\langle i,j \rangle} \sigma_i \sigma_j$$

The Tc was evaluated through magnetization, magnetic susceptibility, internal energy and specific heat. The lattices were simulated by Monte Carlo method using Wolff cluster algorithm. Furthermore, we used known results of non-fractals lattices as adjuster to the simulated fractals.

The observables were obtained:

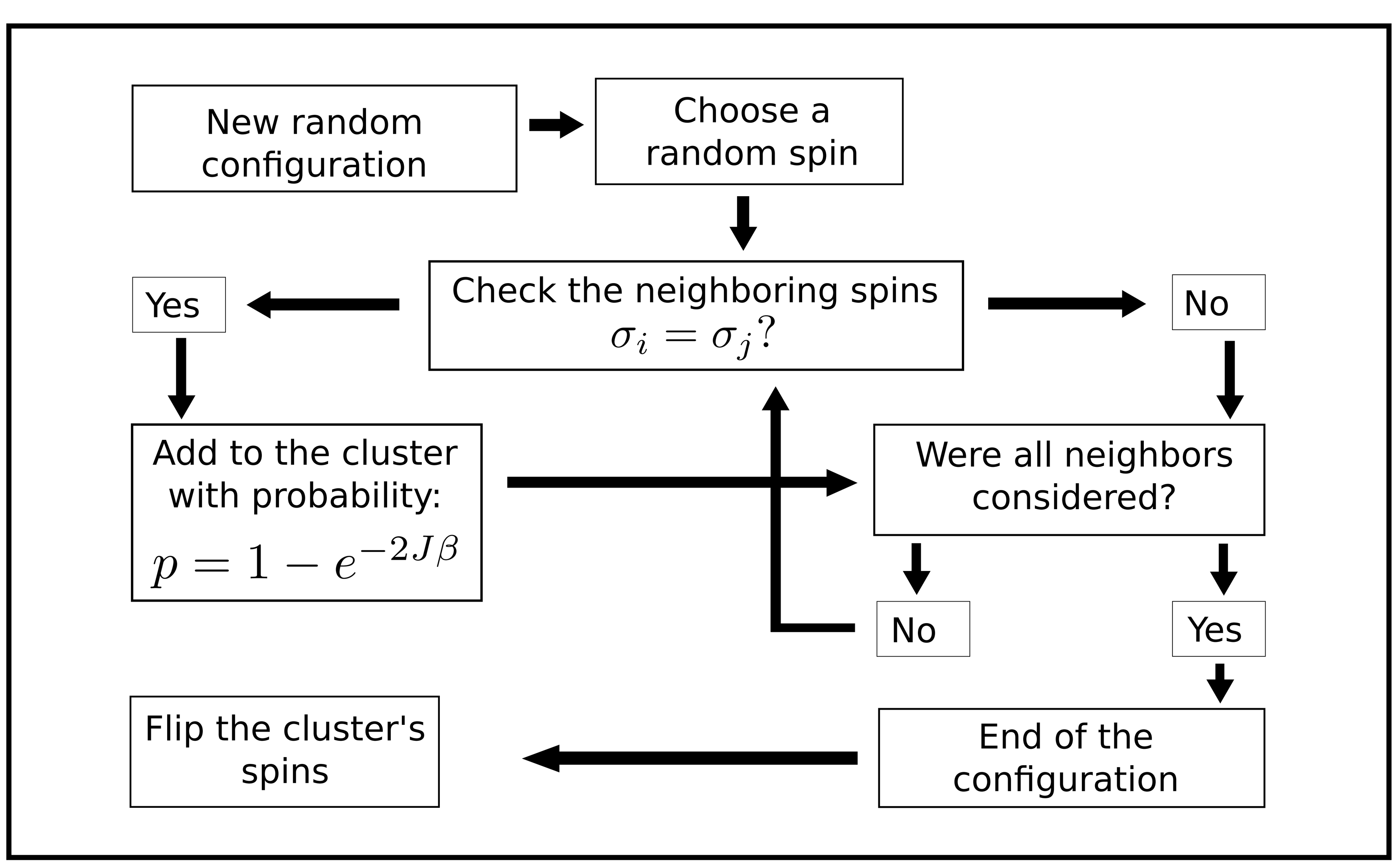
$$\langle O \rangle = \sum_{i=1}^{n} \frac{O_i e^{-\beta H_i}}{Z} \qquad Z = \sum_{i=1}^{n} e^{-\beta H_i}$$

Magnetic suscetibility and specific heat:

$$\chi = \beta[\langle m^2 \rangle - \langle m \rangle^2]$$

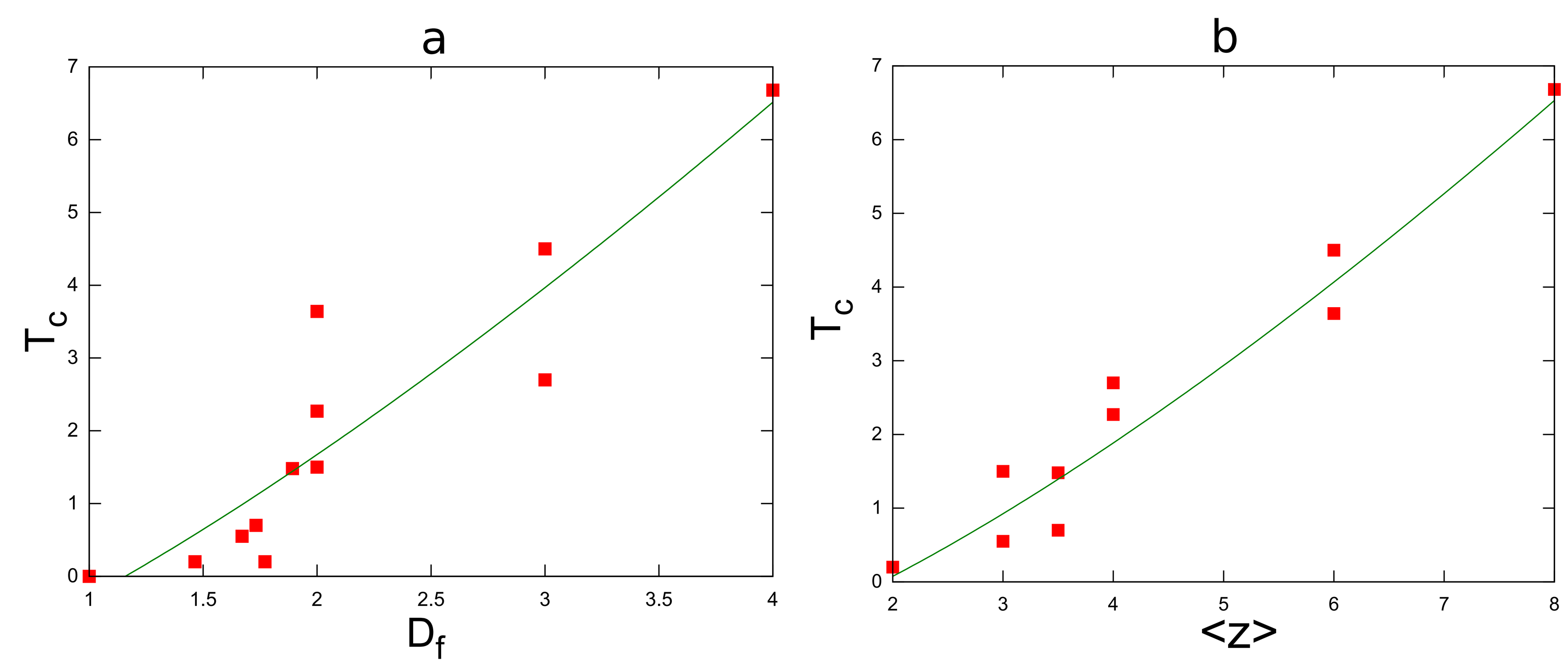
$$C = \beta^2[\langle U^2 \rangle - \langle U \rangle^2]$$

The Wolff cluster algorithm consists in evaluate the magnetic behavior of the lattice using a cluster flipping system. This algorithm is summarized in the diagram below:



RESULTS

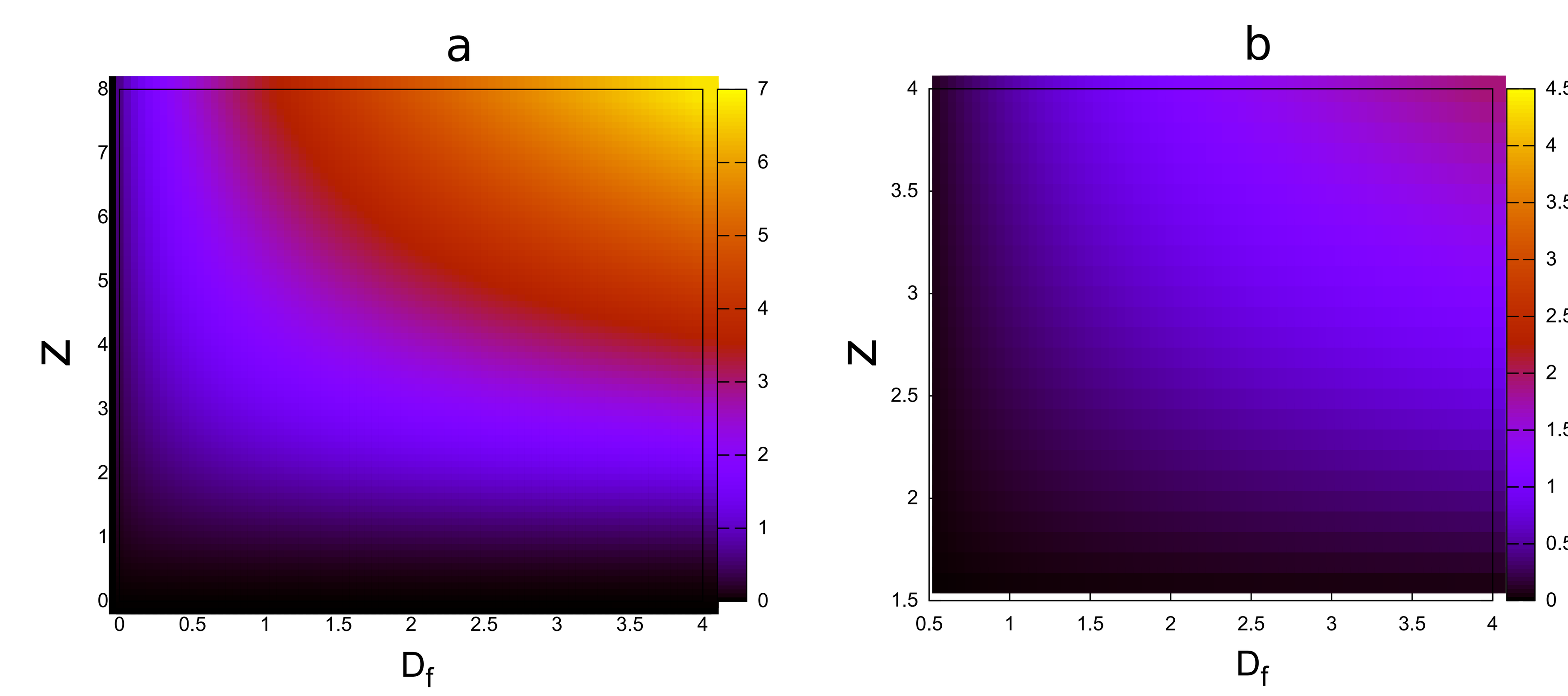
The obtained results were, firstly, evaluated separately to each topological feature: fractal dimension and average coordination number, as follows:



Images 2a and 2b - Fractal dimension (a) and average coordination number (b) with respect to Tc.

With these topological features considered, we can see the dependence among the Tc and its geometric properties. Lacunarity quantifies the emptiness of a lattice. It is used to distinguish two objects with the same D_f and z. These all features were used to describe a general law for the critical temperature:

$$T_c = (1.8D^{0.6} + 0.4z^{2.6} - 1.2L^{0.5})^{1.2} - (1.8D^{0.6})^{1.2} - (0.4z^{2.6})^{1.2} + (1.2L^{0.5})^{1.2}$$



Images 2a e 2b - Relation beteween Df and Tc for L=0(a) e L=1(b).

DISCUSSION

The relation between geometry and nature was considered by Mandelbrot since the 60's[7]. However, this connection is not yet very understood. Here we presented how geometric characteristics can be related to the magnetic critical behavior. This results suggests a strong influence between temperature and the topological features of every lattice.

Despite the coeficients of the presented equation are not yet very clear, this results were expected since D_f and z are related to the amount of interactions among the spins.

Fractals expand the geometric viewpoint of criticality, possibiliting new configurations to study. Its presence in the natural world suggests a deep understanding that has to be considered.

REFERENCES

[1] Kadanoff, L.P., J. Stat. Phys. 137, 777-797 (2009).
[2] Brush, S.G., Rev. Mod. Phys. 39, 883-893 (1967).
[3] Marin, J.J., Magnetic Phase Transition of the Viscek Fractal, Int. J. Phys. Math. Sciences, Vol 3, No 1, 152-170 (2012).
[4] Hisiao, P., Critical Behavior of the Ferromagnetic Ising Model on a Sierpinski Carpet: Monte Carlo Renormalization Group Study. Phys. Rev. B 67, 64411 (2003).
[5] Hisiao, P., Critical exponents of ferromagnetic Ising model on fractal lattices. Symmetry and Structural Properties of Condensed Matter, 318-312 (2001).
[6] Mandelbrot, B.B., The fractal geometry of nature. W.H. Freeman and Company, New York,(1982).
[7] Gefen, Y., Mandelbrot, B.B., and Aharony, A., Phys. Rev. Lett. 45, 855-858 (1980).